EXHIBIT L

Robert V. Hogg
Allen T. Craig
THE UNIVERSITY OF IOWA

Introduction to Mathematical Statistics

Fourth Edition

Macmillan Publishing Co., Inc.
NEW YORK
Collier Macmillan Publishers
LONDON

Printed in the United States of America

Macmillan Publishing Co., Inc.
866 Third Avenue, New York, New York 10022

Collier Macmillan Canada, Ltd.

Library of Congress Cataloging in Publication Data

Hogg, Robert V
Introduction to mathematical statistics.

Bibliography: p.
Includes index.
1. Mathematical statistics. I. Craig, Allen Thornton, (date) joint author. II. Title.
QA276.H59 1978 519 77-2884
ISBN 0-02-355710-9 (Hardbound)
ISBN 0-02-978990-7 (International Edition)

PRINTING 22 23 24 25 26 27 28 YEAR 2 3 4 5

ISBN 0-02-355710-9

and

$$E\left\{\exp\left[t\left(\frac{X-\mu}{\sigma}\right)\right]\right\} = e^{-\mu t/\sigma}M\left(\frac{t}{\sigma}\right), \qquad -h\sigma < t < h\sigma.$$

1.90. Show that the moment-generating function of the random variable X having the p.d.f. $f(x) = \frac{1}{3}$, $-1 < x < 2$, zero elsewhere, is

$$M(t) = \frac{e^{2t} - e^{-t}}{3t}, \qquad t \neq 0,$$

$$= 1, \qquad t = 0.$$

1.91. Let X be a random variable such that $E[(X - b)^2]$ exists for all real b. Show that $E[(X - b)^2]$ is a minimum when $b = E(X)$.

1.92. Let $f(x_1, x_2) = 2x_1$, $0 < x_1 < 1$, $0 < x_2 < 1$, zero elsewhere, be the p.d.f. of X_1 and X_2. Compute $E(X_1 + X_2)$ and $E\{[X_1 + X_2 - E(X_1 + X_2)]^2\}$.

1.93. Let X denote a random variable for which $E[(X - a)^2]$ exists. Give an example of a distribution of a discrete type such that this expectation is zero. Such a distribution is called a *degenerate distribution*.

1.94. Let X be a random variable such that $K(t) = E(t^X)$ exists for all real values of t in a certain open interval that includes the point $t = 1$. Show that $K^{(m)}(1)$ is equal to the mth *factorial moment* $E[X(X - 1)\cdots(X - m + 1)]$.

1.95. Let X be a random variable. If m is a positive integer, the expectation $E[(X - b)^m]$, if it exists, is called the mth moment of the distribution about the point b. Let the first, second, and third moments of the distribution about the point 7 be 3, 11, and 15, respectively. Determine the mean μ of X, and then find the first, second, and third moments of the distribution about the point μ.

1.96. Let X be a random variable such that $R(t) = E(e^{t(X-b)})$ exists for $-h < t < h$. If m is a positive integer, show that $R^{(m)}(0)$ is equal to the mth moment of the distribution about the point b.

1.97. Let X be a random variable with mean μ and variance σ^2 such that the third moment $E[(X - \mu)^3]$ about the vertical line through μ exists. The value of the ratio $E[(X - \mu)^3]/\sigma^3$ is often used as a measure of *skewness*. Graph each of the following probability density functions and show that this measure is negative, zero, and positive for these respective distributions (said to be skewed to the left, not skewed, and skewed to the right, respectively).

(a) $f(x) = (x + 1)/2$, $-1 < x < 1$, zero elsewhere.
(b) $f(x) = \frac{1}{2}$, $-1 < x < 1$, zero elsewhere.
(c) $f(x) = (1 - x)/2$, $-1 < x < 1$, zero elsewhere.