

EXHIBIT 17

BASIC ECONOMETRICS

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CHAPTER 6

EXTENSIONS OF THE TWO-VARIABLE LINEAR REGRESSION MODEL

(6)

(7)

$E(u_0)$

is zero by assumption

get $\text{var}(Y_0 - \hat{Y}_0) =$
variance and covariance
at $\text{var}(u_0) = \sigma^2$, we

= (5.10.6)

Some aspects of linear regression analysis can be easily introduced within the framework of the two-variable linear regression model that we have been discussing so far. First we consider the case of **regression through the origin**, that is, a situation where the intercept term, β_1 , is absent from the model. Then we consider the question of the **units of measurement**, that is, how the Y and X variables are measured and whether a change in the units of measurement affects the regression results. Finally, we consider the question of the **functional form** of the linear regression model. So far we have considered models that are linear in the parameters as well as in the variables. But recall that the regression theory developed in the previous chapters only requires that the parameters be linear; the variables may or may not enter linearly in the model. By considering models that are linear in the parameters but not necessarily in the variables, we show in this chapter how the two-variable models can deal with some interesting practical problems.

Once the ideas introduced in this chapter are grasped, their extension to multiple regression models is quite straightforward, as we shall show in Chapters 7 and 8.

6.1 REGRESSION THROUGH THE ORIGIN

There are occasions when the two-variable PRF assumes the following form:

$$Y_i = \beta_2 X_i + u_i \quad (6.1.1)$$

*Population
Regression
Function*

In this model the intercept term is absent or zero, hence the name regression through the origin.

As an illustration, consider the Capital Asset Pricing Model (CAPM) of modern portfolio theory, which, in its risk-premium form, may be expressed as¹

$$(ER_i - r_f) = \beta_i(ER_m - r_f) \quad (6.1.2)$$

where ER_i = expected rate of return on security i

ER_m = expected rate of return on the market portfolio as represented by, say, the S&P 500 composite stock index

r_f = risk-free rate of return, say, the return on 90-day Treasury bills

β_i = the Beta coefficient, a measure of systematic risk, i.e., risk that cannot be eliminated through diversification. Also, a measure of the extent to which the i th security's rate of return moves with the market. A $\beta_i > 1$ implies a volatile or aggressive security, whereas a $\beta_i < 1$ a defensive security. (Note: Do not confuse this β_i with the slope coefficient of the two-variable regression, $\beta_{2.1}$.)

If capital markets work efficiently, then CAPM postulates that security i 's expected risk premium ($= ER_i - r_f$) is equal to that security's β coefficient times the expected market risk premium ($= ER_m - r_f$). If the CAPM holds, we have the situation depicted in Fig. 6.1. The line shown in the figure is known as the **security market line (SML)**.

For empirical purposes, (6.1.2) is often expressed as

$$R_i - r_f = \beta_i(R_m - r_f) + u_i \quad (6.1.3)$$

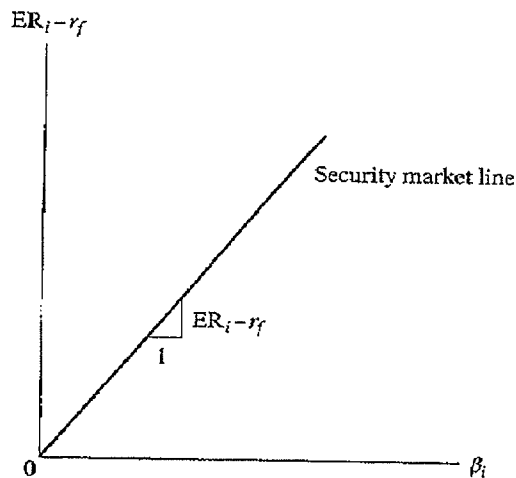


FIGURE 6.1
Systematic risk.

¹See Haim Levy and Marshall Sarnat, *Portfolio and Investment Selection: Theory and Practice*, Prentice-Hall International, Englewood Cliffs, N.J., 1984, Chap. 14.

or

$$R_i - r_f = \alpha_i + \beta_i(R_m - r_f) + u_i \quad (6.1.4)$$

The latter model is known as the **Market Model**.² If CAPM holds, α_i is expected to be zero. (See Fig. 6.2.)

In passing, note that in (6.1.4) the dependent variable, Y , is $(R_i - r_f)$ and the explanatory variable, X , is β_i , the volatility coefficient, and *not* $(R_m - r_f)$. Therefore, to run regression (6.1.4), one must first estimate β_i , which is usually derived from the **characteristic line**, as described in exercise 5.5. (For further details, see exercise 8.34.)

As this example shows, sometimes the underlying theory dictates that the intercept term be absent from the model. Other instances where the zero-intercept model may be appropriate are Milton Friedman's permanent income hypothesis, which states that permanent consumption is proportional to permanent income; cost analysis theory, where it is postulated that the variable cost of production is proportional to output; and some versions of monetarist theory that state that the rate of change of prices (i.e., the rate of inflation) is proportional to the rate of change of the money supply.

How do we estimate models like (6.1.1), and what special problems do they pose? To answer these questions, let us first write the SRF of (6.1.1), namely,

$$Y_i = \hat{\beta}_2 X_i + \hat{u}_i \quad (6.1.5)$$

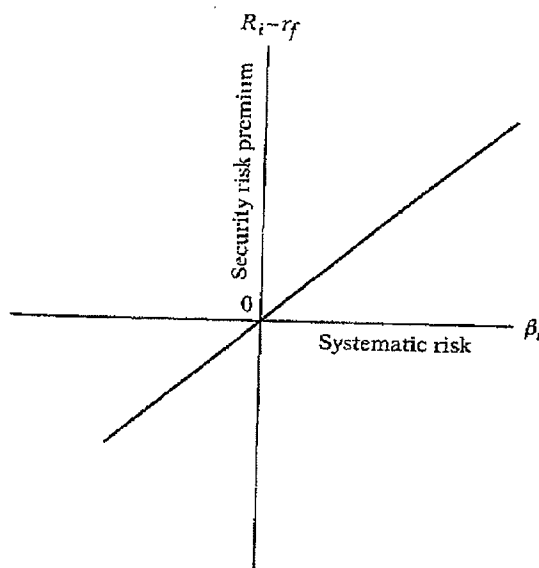


FIGURE 6.2
The Market Model of Portfolio Theory
(assuming $\alpha_i = 0$).

²See, for instance, Diana R. Harrington, *Modern Portfolio Theory and the Capital Asset Pricing Model: A User's Guide*, Prentice-Hall, Englewood Cliffs, N.J., 1983, p. 71.