EXHIBIT 4

Marketing Models and Econometric Research

Leonard J. Parsons
Claremont Graduate School

Randall L. Schultz
Purdue University

NORTH HOLLAND · NEW YORK
NEW YORK • OXFORD

Elsevier North Holland, Inc.
52 Vanderbilt Avenue, New York, N.Y. 10017

Distributors outside the United States and Canada:

Thomond Books
(A Division of Elsevier/North Holland Scientific Publishers, Ltd.)
P.O. Box 85
Limerick, Ireland

Second Printing, 1978

Library of Congress Cataloging in Publication Data

Parsons, Leonard J.
Marketing models and econometric research.

Bibliography: p.
Includes index.
1. Marketing–Mathematical models. 2. Marketing research–Mathematical models. I. Schultz, Randall L., joint author. II. Title.
HF5415.P2494 658.8'007'2 76-476
ISBN 0-7204-8601-7

Manufactured in the United States of America.

models according to some criterion. The criterion can either be an informal decision rule such as maximizing $\bar{R}^2$ or a formal decision rule involving hypothesis testing.

Informal Decision Rule

The most common decision rule for choosing among alternative linear models with nonstochastic exogenous variables is to select the model with the largest $\bar{R}^2$. $\bar{R}^2$ is an adjustment of R^2, the coefficient of determination:

$$\bar{R}^2 = R^2 - \frac{K-1}{n-K}(1 - R^2), \tag{5.21}$$

where K is the number of exogenous variables and n is the number of observations. The adjustment approximately corrects for the bias caused by the fact that R^2 can be increased simply by adding more variables. An equivalent rule is select the model with the smallest residual variance. Moreover, the probability that the decision rule will choose a particular model when it is the correctly specified model can be calculated. Ideally, this probability should be large.

The problem is to choose between two alternative models:

$$\text{(A1)} \qquad \mathbf{y} = \mathbf{X}\beta + \epsilon$$

and

$$\text{(A2)} \qquad \mathbf{y} = \mathbf{Z}\gamma + \eta.$$

The number of independent variables in $\mathbf{X}$ is K_x and in $\mathbf{Z}$ is K_z. The number of observations is n. The dependent variable, $\mathbf{y}$, is assumed to have a multivariate normal distribution with mean vector ξ and covariance matrix $\sigma^2 \mathbf{I}_n$. The probability of choosing model A1 over model A2 by the maximum $\bar{R}^2$ criterion when model A1 is the correctly specified model is $\Pr(\bar{R}_x^2 > \bar{R}_z^2) = \Pr(\mathbf{y}'\mathbf{A}\mathbf{y} < 0)$. The symmetric matrix $\mathbf{A}$ is defined as $\mathbf{M}_x - \alpha\mathbf{M}_z$, where $\mathbf{M}_x = \mathbf{I}_n - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$, $\mathbf{M}_z = \mathbf{I}_n - \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$, and $\alpha = (n - K_x)/(n - K_z)$. This result is due to Schmidt (1973) and Ebbeler (1974).

Imhof (1961) developed a procedure for calculating the distribution of quadratic forms in normal variables. The Imhof procedure can be implemented by modification of a computer program given in Koerts and Abrahamse (1969). This approach was utilized in a marketing study by Parsons (1976).

There are several good uses for goodness of fit, especially as measured by $\bar{R}^2$. It is a particularly appropriate measure of the extent to which the (true) model accounts for total variation or, in a sense, approximates the real phenomena. In this case, $\bar{R}^2$ is a measure of the degree of approximation by which a generalization holds. For testing theories, however, a more powerful criterion is necessary. These ideas do not appear to be widely understood in marketing or economics as has been argued, notably by Bass (1969).

The major weakness of this criterion is rather fundamental: It doesn't work when none of the alternative specifications are correct. Moreover, since it only holds "on the average" there is no small chance that the wrong specification prevails. Now since the objective of marketing econometrics is to identify true sales response functions and marketing decision rules, a criterion which relies on the fortuitous inclusion of the correct model as one of the set under evaluation seems to be an inefficient way of conducting research. It is inefficient because the process does not encourage the development of specification (models) which are otherwise readily falsifiable, and therefore incorrect models are more easily accepted.

Moreover, this criterion assumes that the dependent variables of the models are identical. Sometimes, however, a marketing researcher may want to explore regressions with sales, Q and $\log Q$ as alternative forms of the dependent variable. In this case, the $\bar{R}^2$ criterion does not obtain directly. In comparing a set of linear market-share models with a set of loglinear models, Weiss (1969) has used antilog conversions and then correlation of y with $\hat{y}$ to evaluate relative goodness of fit.

Hypothesis Testing

The maximum $\bar{R}^2$ selection rule discussed in the last section is a methodological convention. It involves an implicit assumption that disagreement between the theoretical model and observations is a monotone decreasing function of $\bar{R}^2$. However, this convention can be in conflict with classical statistical inference. In classical statistical inference, the disagreement between the theoretical model and observations is a monotone decreasing function of the probability with which the observed event is supposed to occur.

The two conventions necessarily yield similar conclusions only if the *population* coefficient of determination, P^2, is equal to 1.0. The probability density function of the *sample* coefficient of determination is noncentral F, with $K - 1$, $n - K$ degrees of freedom and noncentrality parameter nP^2. This reduces to the familiar central F (5.4) when the null hypothesis is $P^2 = 0$.

Basmann (1964) provides the following illustration of this distinction. Suppose we are able to derive, from a conjunction of the underlying behavioral marketing postulates and the given sample observations of size 20 on 3 exogenous variables, a statement that P^2 just lies between 0.3 and 0.4. Furthermore, suppose we obtain $R^2 = .75$ in our regression run. Under the first convention, we may well judge that this test statistic does not disagree with our model. However, since

$$\int_{.70}^{1.0} f(R^2; 20P^2; 2)d(R^2) \leqslant 0.05,$$

under the second convention, we would decide that the observed sample coefficient of determination is *too large* to be in good agreement with our marketing postulates.

Embedded Alternatives. One way to discriminate among linear models is to embed the specific alternative models within one general model. Hypotheses about the values of certain parameters of the general model can be deduced from the specific alternative models. For instance, suppose we want to choose between these two alternative specific models

$$y_\alpha = \beta_0 + \beta_1 x_\alpha + \beta_2 x_{\alpha-1} + \epsilon_\alpha, \tag{5.22}$$

and

$$y_\alpha = \beta_0 + \beta_1 x_\alpha + \beta_3 y_{\alpha-1} + \epsilon_\alpha. \tag{5.23}$$

Then, we can embed these two models in this general model

$$y_\alpha = \beta_0 + \beta_1 x_\alpha + \beta_2 x_{\alpha-1} + \beta_3 y_{\alpha-1} + \epsilon_\alpha. \tag{5.24}$$