EXHIBIT 15

ECONOMETRIC MODELS AND ECONOMIC FORECASTS

Second Edition

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McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

In this section we deal with the problem of *first-order serial correlation*, in which errors in one time period are correlated directly with errors in the ensuing time period.† While it is certainly possible that serial correlation can be negative as well as positive, we concern ourselves primarily with the case of positive serial correlation, in which errors in one time period are positively correlated with errors in the next time period. Positive serial correlation frequently occurs in time-series studies, either because of correlation in the measurement error component of the error term or more likely, because of the high degree of correlation over time present in the cumulative effects of the omitted variables in the regression model.

As a general rule, the presence of serial correlation will not affect the unbiasedness or consistency of the ordinary least-squares regression estimators, but it does affect their efficiency.‡ In the case of positive serial correlation, this loss of efficiency will be masked by the fact that the estimates of the standard errors obtained from the least-squares regression will be smaller than the true standard errors. In other words, the regression estimators will be unbiased, but the standard error of the regression will be biased downward.§ This will lead to the conclusion that the parameter estimates are more precise than they actually are. There will be a tendency to reject the null hypothesis when, in fact, it should not be rejected. We shall not prove these results in the chapter, but one can obtain an intuitive feeling for why they are true by examining Fig. 6.1*a* and *b*.¶

Both graphs illustrate the presence of positive serial correlation in a model with a single independent explanatory variable. In Fig. 6.1*a*, the error term associated with the first observation happens to be positive. This leads to a series of error terms, the first four of which are positive and the last two of which are negative. In Fig. 6.1*b*, the oppositive case has occurred, the first four errors being negative, and the last two being positive. In the first case the estimated regression slope is lower than the true slope, while in the second case it is higher. Since both cases are equally likely to occur, it seems reasonable that least-squares slope estimates will be correct on average; i.e., they will be unbiased. However, in both cases, the least-squares regression lines fit the observed sample data points more closely than the true regression line; this leads to an R^2 that gives an overly optimistic picture of the success of least-squares regression. More important, however, least squares will lead to an estimate of the error variance which is smaller than the true error variance.|| Once again the success of the regression procedure will be overstated if the least-squares estimate of the error variance is used to do statistical tests.

† The more general case can be handled with the use of generalized least-squares estimation, as detailed in Appendix 6.1 and with time-series techniques discussed in Part Three.

‡ If the model includes a lagged dependent variable, the problems are much more severe. The lagged-dependent-variable case will be discussed briefly in Section 7.7.

§ This holds provided that the X's are not negatively serially correlated.

¶ Some of these assertions are proved in Appendix 6.1.

|| For a discussion of the lack of bias and consistency of the parameter estimators as well as details concerning efficiency, see Kmenta, op. cit., sec. 8-2.

where

$$Y_t^* = Y_t - \rho Y_{t-1} \qquad X_{2t}^* = X_{2t} - \rho X_{2t-1}$$
$$X_{kt}^* = X_{kt} - \rho X_{kt-1} \qquad v_t = \varepsilon_t - \rho\varepsilon_{t-1}$$

are *generalized differences* of Y_t, $X_{2t}, \ldots, X_{kt}$, and v_t. By construction the transformed equation has an error process which is independently distributed with 0 mean and constant variance [see Eq. (6.12)]. Thus, ordinary least-squares regression applied to Eq. (6.19) will yield efficient estimates of all the regression parameters. Of course, the intercept of the original model must be calculated from the estimated intercept associated with Eq. (6.19).†

We have restricted our discussion of serial correlation to the case in which ρ is strictly less than 1. However, the case in which ρ is identically equal to 1 is of particular interest because it leads to a commonly used estimation procedure.‡ The solution process, known as *first-differencing*, is applied if we estimate the transformed equation (by analogy to the generalized differencing procedure):

$$Y_t^* = \beta_2 X_{2t}^* + \beta_3 X_{3t}^* + \cdots + \beta_k X_{kt}^* + v_t \qquad \text{where } Y_t^* = Y_t - Y_{t-1} \quad (6.20)$$
$$X_{2t}^* = X_{2t} - X_{2t-1}$$
$$X_{kt}^* = X_{kt} - X_{kt-1}$$
$$v_t = \varepsilon_t - \varepsilon_{t-1}$$

Note that first-differencing eliminates the need for a constant term in the transformed equation. The intercept of the original equation must be calculated by solving in the original equation when the variables are measured at their respective means.§ If a constant term were included, it would pick up the effect of any time trend present in the initial model.

The generalized differencing procedure would be very useful if the value of ρ were known *a priori*. Because this is usually not the case, we examine three alternative procedures for estimating ρ, each of which has certain computational advantages and disadvantages. All three of these procedures yield estimated parameters with the desired properties when the sample size is large, but little is known about their small-sample properties.

† There is only one serious difficulty associated with the generalized differencing process. As described, the transformed equation is defined only for the time period 2, 3, . . . , T. Dropping the initial time period from the regression procedure seems plausible, but it results in the loss of important information. A better solution would take the first time period observations into account as follows:

$$Y_1^* = \sqrt{1-\rho^2}\, Y_1 \qquad X_{21}^* = \sqrt{1-\rho^2}\, X_{21} \qquad \cdots \qquad X_{k1}^* = \sqrt{1-\rho^2}\, X_{k1}$$

This transformation works because it adjusts the variance of Y and the X's for the first time period, so that the corresponding error variance is equal to the error variance associated with all other time periods. By construction, $\varepsilon_1^* = (1-\rho^2)^{1/2}\varepsilon_1$ and $\text{Var}(\varepsilon_1^*) = (1-\rho^2)\text{Var}(\varepsilon_1) = \sigma_v^2$.

‡ Note, however, that as ρ approaches 1, the error variance in the original equation becomes infinitely large, so that the previous analysis does not follow.

§ In the two-variable model, for example, $Y_t^* = \beta X_t^*$. To obtain the intercept estimate, we estimate β and then substitute to obtain $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$.