

Exhibit 3

Proving Antitrust Damages

Legal and
Economic Issues

Second Edition



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CHAPTER 6

ECONOMETRICS AND REGRESSION ANALYSIS

A. Introduction to Econometrics and Regression Analysis

Damages analyses require estimates of the impact of an allegedly anticompetitive act on specific market outcomes. However, market outcomes often result from a complex interaction among a large number of factors. For example, once liability has been established in an antitrust price-fixing case, the questions are (1) whether the alleged conspiracy affected prices (the fact of injury) and, if so, (2) the extent to which prices have increased as a result of the alleged conspiracy (the amount of injury).¹ Answering these questions is complicated by the fact that the prices prevailing in the market likely were affected by a wide variety of other demand and supply factors unrelated to the alleged conspiracy. Isolating and measuring the effect of the alleged conspiracy on price requires properly accounting for these other factors. Otherwise, one might confuse the effects of one or more of these other factors with the effects of the alleged conspiracy. Econometric and regression analyses are particularly useful in separating the impact of an alleged anticompetitive act on market outcomes (such as pricing) from the impact of other influences.²

Econometric techniques are statistical methods designed to uncover the relationship between a dependent variable (the object of the analysis, such as prices) and one or more explanatory variables (the potential influences on the dependent variable, such as price fixing, costs, or demand factors). In particular, econometric techniques can provide an estimate of the partial effect of each of the explanatory variables on the dependent variable. An explanatory variable's partial effect is the change in the dependent variable that would result from a change in that explanatory variable, holding all of the other explanatory variables

1. See John E. Lopatka, *Antitrust Injury and Causation*, in III ABA SECTION OF ANTITRUST LAW, ISSUES IN COMPETITION LAW & POLICY 2299, 2311 (2008); Theon van Dijk & Frank Verboven, *Quantification of Damages*, in III ABA SECTION OF ANTITRUST LAW, ISSUES IN COMPETITION LAW & POLICY 2331, 2334 (2008).
2. See Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods in Antitrust: Review and Critique*, 1 AMER. L. & ECON. REV. 386, 388 (1999).

constant. Thus, when correctly implemented, econometric techniques can isolate and measure the effect of a single explanatory factor on the economic outcomes that are relevant when estimating damages.³

Even sophisticated econometric techniques are of no use if the partial effect of interest is not identified (i.e., distinguishable).⁴ Identification is often a function of the available data. In many situations, identification requires that the existence (or the extent) of the alleged unlawful conduct varied over the observations in the data in a way that is not perfectly mirrored by ("correlated" with) other explanatory variables. If such data exist and there are adequate controls for the effects of other economic factors, identification generally will be possible.⁵

Assuming liability and given the construction of a reasonable explanatory variable to represent the anticompetitive act, properly applied econometric techniques can provide a reliable estimate of damages based on a statistical estimate of the partial effect of the anticompetitive act on the dependent variable of interest, such as price. These techniques also can provide useful information about the important damage-related issue of causation, which is linking the assumed anticompetitive acts to adverse outcomes for competition and for plaintiffs. In particular, econometric analyses can test whether the data are consistent or inconsistent with the proposition that the relevant explanatory variable "caused" changes in the dependent variable.⁶ However, econometric analysis cannot demonstrate causality in the strictest sense as can a randomized experimental design, such as that used in laboratory experiments.⁷

One frequently used econometric methodology is the before-during approach. It can be used when data exist from both the period before an alleged anticompetitive act (e.g., a price-fixing conspiracy) and the period of the alleged anticompetitive act. This approach identifies the effect of an alleged anticompetitive act by comparing, for example, prices in the "during" period (the affected period) to prices in the "before" period (the control period), while also controlling for differences in other economic factors between the during and before periods. "Before-during" is used here to distinguish it from a "before-during-after" approach. This latter approach can be used when data exist

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3. See JEFFREY M. WOOLDRIDGE, *ECONOMETRIC ANALYSIS OF CROSS SECTION AND PANEL DATA* 3-4 (2002).
 4. *Id.* at 52-53.
 5. *Id.*
 6. *Id.* at 3-4. See also Chapter III.C.
 7. See WOOLDRIDGE, *supra* note 3, at 3-4.

not only from before and during, but also from after, the alleged unlawful conduct.⁸

Another frequently employed methodology uses data from a *benchmark* or *yardstick* market,⁹ where the alleged unlawful conduct did not occur, as a control group.¹⁰ A benchmark market may be a different area than where the unlawful conduct occurred, a different product, or a different end use market. The idea again is that one can compare the prices or profit margins in the market of interest (the *treatment* market) to prices in the benchmark (control) market, accounting for other economic factors that might have differed between the two markets. This approach may be particularly useful when the data available from the time before the anticompetitive act are limited or unreliable.¹¹

Sometimes the benchmark and before-during approaches can be combined into what is called the *difference-in-differences* approach.¹² Under this approach, the before period is used to calibrate the difference between the variable of interest (e.g., price) in the treatment and benchmark markets in the absence of the alleged unlawful conduct. To identify the partial effect of the alleged unlawful conduct, this approach then subtracts the difference between the variable (price) in the market at issue and in the benchmark market in the during period from that difference in the before period (i.e., the difference in the differences).¹³ For example, suppose prices were \$10 higher on average in the affected market than in the benchmark market before the alleged conduct, but that differential increased to \$15 during the alleged conduct, controlling for

8. See van Dijk & Verboven, *supra* note 1, at 2335.

9. Unless otherwise noted, our use of the term "market" is meant to denote an economic market, and not necessarily an antitrust relevant market.

10. For a discussion of benchmark approaches see, e.g., PHILLIP E. AREEDA ET AL., *ANTITRUST LAW: AN ANALYSIS OF ANTITRUST PRINCIPLES AND THEIR APPLICATION* 483 (2d ed. 2005); Robert H. Lande & James Langenfeld, *The Perfect Capex? Private Damages and the Microsoft Case*, 69 *GEO. WASH. L. REV.* 902, 903, 907, 908-09 (2001). A benchmark approach or before-during-after approach is similar to a natural experiment, in that both rely on a control group or event that is economically similar to the market being studied, but is unaffected by the alleged anticompetitive act. See Mary Coleman & James Langenfeld, *Natural Experiments*, in *I ABA SECTION OF ANTITRUST LAW, ISSUES IN COMPETITION LAW & POLICY* 743 (2008).

11. For a discussion of benchmark analyses, see part F of this chapter.

12. See WOOLDRIDGE, *supra* note 3, at 130.

13. *Id.* One might still want to control for other economic factors that might have changed between the two periods in the two markets. *Id.*

changes in other economic factors. In this example, an estimate of the effect of the alleged conduct would be a \$5 price increase.

When undertaking an econometric analysis, it is important to recognize that data need not be perfect to be reliable. For example, it is not necessary to observe data on every factor that affected the dependent variable. Indeed, if the available data were that complete, econometric methods may not be needed.¹⁴ Nor is it necessary to have perfectly measured variables.¹⁵ Nevertheless, the statistical precision of the econometric results generally will be greater the more accurate and plentiful the available data, and the required econometric techniques generally will be less complex with better quality data.

With sufficiently reliable data and identification, economists have a wide variety of econometric techniques available to them to estimate the partial effect of the alleged unlawful conduct in an antitrust case.¹⁶ The appropriate technique to use in a given circumstance depends on the type of available data, the properties of the underlying economic variables, and the specification of the econometric model used.

This chapter covers a wide range of issues. Not all of the tests or analyses discussed here need to be performed in every damages analysis, and the ones that are important for a particular case will depend on the specific facts of the case.

B. Legal Requirements

The legal requirements for regression analysis fall under the rules for testimony by experts. Under Federal Rule of Evidence 702:

If scientific, technical, or other specialized knowledge will assist the trier of fact to understand the evidence or to determine a fact in issue, a witness qualified as an expert by knowledge, skill, experience, training,

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14. See G.S. MADDALA, *INTRODUCTION TO ECONOMETRICS* 5 (3d ed. 2001).
 15. See Jerry A. Hausman, *Mismeasured Variables: Problems From the Right and Problems From the Left*, 15 *J. ECON. PERSPECTIVES* 57 (2001), for a discussion of measurement error, and WOOLDRIDGE, *supra* note 3, at 63-64, for a discussion of the use of "proxy" variables.
 16. Such techniques include ordinary least squares regression, nonlinear least squares regression, maximum likelihood, instrumental variables estimators, generalized method of moments estimators, and various semi-parametric and nonparametric estimators. For a description of a wide range of econometric techniques, see generally WOOLDRIDGE, *supra* note 3, and WILLIAM GREENE, *ECONOMETRIC ANALYSIS* (4th ed. 2000).

or education, may testify thereto in the form of an opinion or otherwise, if (1) the testimony is based upon sufficient facts or data, (2) the testimony is the product of reliable principles and methods, and (3) the witness has applied the principles and methods reliably to the facts of the case.¹⁷

Regression analyses have met these requirements many times in litigation for a wide range of issues, including the estimation of antitrust damages.¹⁸ A competent expert should be able to tell the litigant whether sufficient facts and data are available for econometric analysis, explain the types of econometric analyses that are applicable, and reliably apply these econometric techniques.

As suggested above, a key advantage of using econometric analysis is that it allows the expert to separate the impact of an anticompetitive act from other economic factors that affect the variable of interest, such as sales volumes and prices. In general, econometric results will be more reliable as the amount and quality of data increase. If sufficient data are available, econometric analysis has been found necessary for achieving the minimum scientific standard for establishing lost sales and price changes. For example, in *Zenith Electronics Corp. v. WH-TV Broadcasting Corp.*,¹⁹ expert opinion and internal forecasts for sales growth were excluded because data to estimate sales growth via regression analysis were available and not used. But using regression analysis does not guarantee that the analysis will be accepted. The regression must be in a form that assists in determining a material fact, such as the amount of lost sales or the size of price changes.²⁰ The analysis also must be based on data "reasonably relied upon by experts in the field."²¹ Because experts typically verify data for accuracy in consulting work or academic research, experts presenting regression

17. FED. R. EVID. 702.

18. See, e.g., *Conwood Co. v. U.S. Tobacco Co.*, 290 F.3d 768 (6th Cir. 2002); *Tuscaloosa v. Harcros Chems.*, 158 F.3d 548 (11th Cir. 1998); *Allapattah Servs. v. Exxon*, 61 F. Supp. 2d 1335 (S.D. Fla. 1999) (order denying motions to preclude and to strike expert testimony).

19. 395 F.3d 416 (7th Cir. 2005).

20. See Daniel Rubinfeld, *Reference Guide on Multiple Regression*, in FEDERAL JUDICIAL CENTER, REFERENCE MANUAL ON SCIENTIFIC EVIDENCE 186 (2d ed. 2000), available at [http://www.fjc.gov/public/pdf.nsf/lookup/sciman03.pdf/\\$file/sciman03.pdf](http://www.fjc.gov/public/pdf.nsf/lookup/sciman03.pdf/$file/sciman03.pdf).

21. FED. R. EVID. 703.

analyses in court need to conduct similar verifications of the data that they use.²²

The principle for reliability encompasses many factors entering the regression analysis, and a competent expert should conduct reliability checks to ensure that the results survive the rigors of litigation. Part D of this chapter discusses many of the issues that arise in implementing reliably econometric techniques and econometric tests that a testifying expert should perform when appropriate. Moreover, econometric results typically should not change materially with minor changes to the data (e.g., deleting a few observations).²³

C. Causation and Quantification

Courts typically require an analysis of damages that not only quantifies the amount of damages, but also demonstrates that those damages are causally linked to the allegedly anticompetitive acts.²⁴ In a price-fixing case, for example, there must be an analysis that provides evidence of a clear link between the agreement to fix prices and an increase in prices that is not explained by other factors (i.e., prices would not have been as high but for the agreement).²⁵

Properly applied econometric techniques can provide both reliable estimates of the magnitude of damages and useful information about causation. As discussed above, assuming liability and the construction of a reasonable explanatory variable representing the anticompetitive act at issue, econometric tests can be used to determine whether the data are consistent or inconsistent with the anticompetitive act having caused changes in the dependent variable.²⁶ Most economic data, however, come from observations, not controlled experiments. Therefore, the economist typically has no control over how the data were generated.

The fact that economic data typically do not come from controlled experiments may present certain challenges to econometric analysis. To see this point, consider an investigator conducting a controlled laboratory experiment. Such an investigator can set the values of the factor of interest and apply different levels to different test subjects. For example, a sample of subjects in a pharmaceutical clinical trial can be assigned

22. See, e.g., *Cooper v. Travelers Indem. Co.*, 113 F. App'x 198 (9th Cir. 2004).

23. See Rubinfeld, *supra* note 20, at 199.

24. See, e.g., Lopatka, *supra* note 1, at 2299.

25. See van Dijk & Verboven, *supra* note 1, at 2335.

26. See WOOLDRIDGE, *supra* note 3, at 3-4.

randomly to a "test" group and a "control" group, with the factor of interest (the pharmaceutical "treatment") applied only to the test group. Given this experimental design, if after treatment, the dependent variable changes in the test group in a way that is not found in the control group, then the treatment very likely caused that change and the impact of the treatment can be estimated.²⁷ Economists rarely have this luxury. Therefore, although econometric techniques applied to observational data can provide evidence consistent with causality and quantification of impact, even sometimes using "control groups" to create natural experiments, econometric analysis typically provides a weaker form of evidence of causality than a randomized experimental design.²⁸ Nevertheless, (1) a properly specified econometric model showing that an explanatory variable has a statistically significant partial effect on the dependent variable, holding constant other factors, and (2) a sound economic theory explaining why one would expect the explanatory variable to have a causal effect, together provide evidence consistent with the existence of a causal relationship and an estimate of the magnitude of the effect.²⁹

For example, assume a properly specified econometric analysis finds a measure of the partial effect of the allegedly anticompetitive act on the dependent variable of interest to be statistically significant (discussed

27. The definition of causality has been the subject of philosophical debate. See John DiNardo, *Interesting Questions in Freakonomics*, 45 J. ECON. LIT. 973 (2007). However, an adequate definition of causality for the purposes of this chapter is that factor A causes outcome B if the random application of factor A in the population would result in outcome B for the treated segment of the population. *Id.*

28. *Id.*

29. See Rubinfeld, *supra* note 20, at 184-85. An econometric test called the *Granger Causality Test* is designed to determine whether changes in one variable occur before changes in another variable. While this property can potentially supply some useful information, it is not the same as causality in the sense in which this chapter uses the term. For example, as PETER KENNEDY, *A GUIDE TO ECONOMETRICS* 64 (6th ed. 2008), notes, Christmas cards "Granger-cause" Christmas, but obviously do not "cause" Christmas. See also Jerry A. Hausman, *Specification and Estimation of Simultaneous Equation Models*, in 1 HANDBOOK OF ECONOMETRICS 391, 435-436 (Z. Griliches & M.D. Intriligator eds., 1983).

below) and of the expected sign.³⁰ This result would be consistent with the act resulting in damages, as long as there is a clear economic explanation of the linkage between the act and the measure of the partial effect. Econometric analysis also can provide an estimate of magnitude of the impact to estimate the amount of damages. A finding that the estimated impact is not statistically different than zero would be consistent with a lack of causation and zero damages.

D. Regression, Hypothesis Testing, and Specification Testing

1. Simple Regression

As discussed above, regression analysis in the context of damage estimation is a statistical method used to analyze the relationship between a dependent variable and a set of explanatory variables based on a sample of observational data. The dependent variable is typically an economic outcome important in estimating any damages sustained by the plaintiff. Examples of such outcomes include price, profit margin, and quantity. The explanatory variables are typically the major economic factors that may explain variation in the dependent variable.³¹

Econometric analyses that reliably estimate the impact of the major influences on the dependent variable begin with econometric modeling. Econometric modeling assumes that the data are generated by underlying economic forces. The econometric model should be designed to be consistent with and incorporate these underlying economic forces.³² In building the econometric model, the economist makes initial modeling or specification choices by employing economic theory and reasoning. This modeling process identifies the likely major influences on the dependent variable and the potential interactions between those influences. Within the model, the impact of a given economic factor on the dependent variable is typically summarized by a parameter or coefficient that can be estimated from the available data.³³ The modeling choices are very important. Deviations of the econometric model specification from the underlying economic process generating the data can lead to specification error, which make the resulting parameter estimates

30. For example, the "expected sign" would be positive if the allegedly competitive act was a price-fixing conspiracy and the dependent variable of interest was price.

31. See KENNEDY, *supra* note 29, at 75.

32. *Id.*

33. See KENNEDY, *supra* note 29, at 3-4.

unreliable—even if they are statistically significant.³⁴ Fortunately, the validity of many specification choices can be tested statistically to ensure that a model is correctly specified. Throughout this chapter, we discuss the rationale for some specification choices and how the validity of specification choices can be statistically tested.

To understand the basics of regression analysis, we begin with the relationship between one dependent and one independent variable, known as a simple regression analysis. Assume that prices are determined by cost, plus some fixed mark-up for profits. Most economic models hypothesize a direct positive relationship between cost and price.³⁵ Although most economic models do not predict a fixed mark-up for profits and they typically identify a number of other factors that influence price, for purposes of exposition we assume this simplified model. The deterministic or mathematical relationship between cost and price can be described by the simple algebraic equation of:

$$PRICE = \beta_0 + \beta_1 COST .$$

Here, the *regression coefficient* β_1 indicates how much price will increase with cost.³⁶ For example, if price (the dependent variable) increases by \$.90 for every dollar that cost (the exogenous variable) increases, then $\beta_1 = .9$. A second regression coefficient β_0 is a *constant term* reflecting a fixed mark up over changes in costs, say \$1. Price will equal the sum of these two effects in this equation. That is if $COST = \$10$, then $PRICE = \$1 + (.9) \times (\$10) = \$10$. If $COST = \$12$, then $PRICE = \$1 + (.9) \times (\$12) = \$11.80$. This relationship is illustrated in Figure 1.

34. See WOOLDRIDGE, *supra* note 3, at 50-51.

35. Jonathan B. Baker & Timothy F. Bresnahan, *Economic Evidence in Antitrust*, in HANDBOOK OF ANTITRUST ECONOMICS 1, 16 (Paolo Buccirossi ed., 2008).

36. See MADDALA, *supra* note 14, at 59-64.

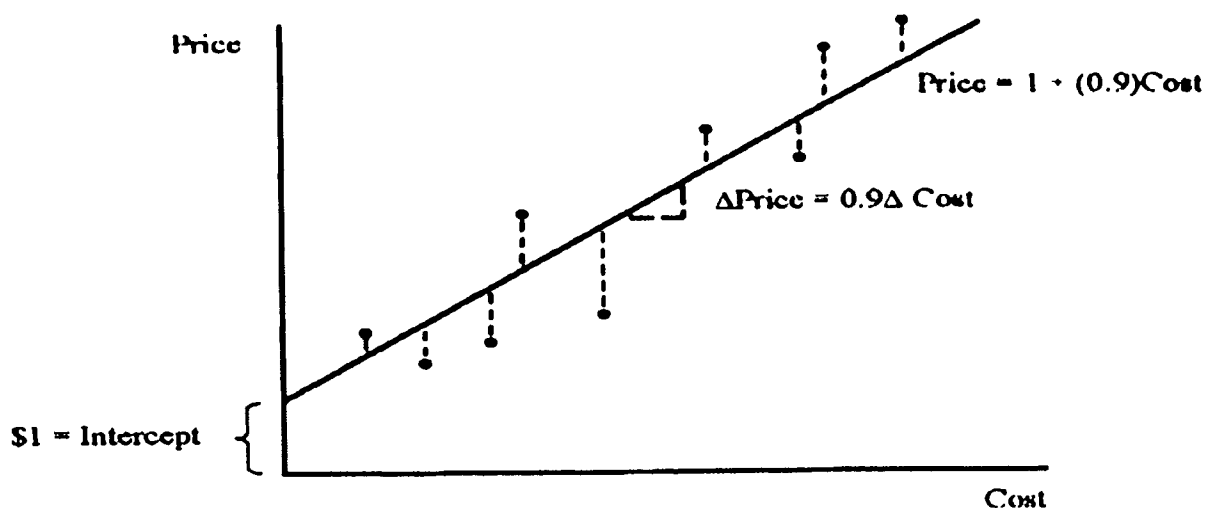


Figure 1

Relationships may be stochastic rather than deterministic. A stochastic relationship is one that has a random component, a component that is not generated from a describable systematic process.³⁷ A stochastic relationship does not yield a single unique value of the dependent variable (e.g., *PRICE*) for a given value of the explanatory variable (e.g., *COST*). In other words, the dependent variable is influenced, but not uniquely determined, by the explanatory variable. The stochastic factors not accounted for by the explanatory variable are summarized in the *error term*, represented by the variable ε in the equation that defines the dependent variable.³⁸ The error term is the difference between the observed dependent variable and the portion of that value that is explained by the explanatory variable. In a simple regression model, this stochastic relationship can be expressed as:

$$PRICE = \beta_0 + \beta_1 COST + \varepsilon.$$

Because the error term is not observed by the econometrician, it amounts to statistical “noise” that partially obscures the “true” underlying relationship between the dependent variable and the included

37. See KENNEDY, *supra* note 29, at 3.

38. *Id.*

explanatory variable.³⁹ Specifically, the statistical noise prevents us from being able to measure the regression coefficients β_0 and β_1 from the data exactly. However, regression analysis provides a way to cut through the statistical noise and obtain *estimates* of each of the regression coefficients, although these estimates will not exactly equal the true underlying coefficients (they are, after all, estimates). Intuitively, one may think of the regression estimate of β_1 as being the average observed impact of cost on price based on the data analyzed, recognizing it is not an exact measure.⁴⁰ Under certain conditions the coefficient estimates will have the desirable property of being *consistent*, which means they converge to the true coefficient values as the size of the data sample grows.⁴¹

One method frequently used for estimating the coefficients that summarize the relationship between the explanatory variables and the dependent variable is *ordinary least squares* (OLS).⁴² This statistical technique minimizes the squared deviations from a line passing through the data, creating the line that fits the pattern of the data as closely as possible. Specifically, it minimizes the sum of the squares of the vertical distances of the observed values to obtain a line that best fits the data, as illustrated in Figure 1.⁴³ This technique has the desirable property of being consistent under general conditions.⁴⁴

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39. The error terms may be caused by unpredictable randomness in the behavior of consumers or businesses, the effect of omitted explanatory variables, or measurement error in the dependent variable. *Id.*
 40. The coefficient is often assumed to be the same for each unit of observation (e.g., customer or firm). However, an economist may want to test this assumption if there is reason to believe it does not hold. See part D.7. of this chapter for further discussion.
 41. See WOOLDRIDGE, *supra* note 3, at 52-54. Coefficient estimates from a regression will be consistent if the error term and the explanatory variables are uncorrelated. *Id.*
 42. Other estimation techniques use different criteria to determine the coefficient estimates. Examples of other techniques include generalized method of moments and maximum likelihood estimation. See WOOLDRIDGE, *supra* note 3, ch. 12-14.
 43. See KENNEDY, *supra* note 29, at 48.
 44. See WOOLDRIDGE, *supra* note 3, at 52-54. OLS moreover will be *unbiased*—correct on average in small samples—under the condition that the conditional expectation of the error term given the explanatory variables is zero. See GREENE, *supra* note 16, at 245. Because the concepts of consistency and unbiasedness have similar meanings for

The relationship in this example assumes that the explanatory variable has a linear (i.e., straight line) relationship with the dependent variable, in that a given change in the dollar value of cost translates to a fixed dollar change in price. However, economic theory, statistical evidence, or information from market behavior may indicate that a different functional form for the regression equation may be more appropriate. For example, in certain instances a given percentage increase in costs will yield a proportionate percent increase in price—rather than a fixed dollar relationship. In this case, a regression equation similar to the one described above can be estimated, but the price and cost data would be transformed into logarithms and the cost coefficient would measure the impact of a percentage change in costs on the percentage change in price.⁴⁵ In general, there should be an economic or statistical basis for the functional form of the regression equation that is estimated.⁴⁶

2. Multiple Regression

As discussed above, one of most compelling reasons for using regression analysis in a study of damages is that multiple regression analysis can help separate the partial effects of various influences on a dependent variable. Multiple regression analysis is a particularly important econometric tool. There typically will be several major influences on a dependent variable, and the econometric model must take these various influences into account in a manner consistent with economic theory and reasoning.

For example, multiple regression analysis might be used to measure the extent to which the price customers paid for a product (PRICE), the dependent variable, is related to three explanatory variables rather than one: (1) the cost of production (COST), (2) the level of industrial production in the downstream industry (DEMAND), and (3) whether the customers purchased the product during the period of an alleged conspiracy (PERIOD).⁴⁷ DEMAND is included because, in addition to

purposes of this chapter, the terms “consistent” and “unbiased” are used interchangeably in this chapter.

45. See KENNEDY, *supra* note 29, at 106.

46. *Id.* at 102.

47. The relevant dependent variable will be determined by the nature of the antitrust allegations, the theory of causation, and specific facts of the case. For example, consider a monopolization case alleging anticompetitive exclusionary conduct through the monopolist knowingly

cost, most economic models predict that the level of demand will affect price. As industry demand increases, prices might be expected to increase. PERIOD is included to account for the alleged anticompetitive act. The allegation in most price-fixing conspiracies is that prices are higher during the conspiracy period, all else equal.⁴⁸

The relationship between the dependent variable and the explanatory variables might take the following form:

$$PRICE = \beta_0 + \beta_1 COST + \beta_2 DEMAND + \beta_3 PERIOD.$$

β_0 , β_1 , β_2 , and β_3 are the regression coefficients associated with the partial effect of each explanatory variable on PRICE (β_0 is, as before, the constant term, β_1 is the coefficient associated with COST, β_2 is the coefficient associated with DEMAND, and β_3 is the coefficient associated with PERIOD).⁴⁹

The coefficient associated with each explanatory variable indicates the amount by which the dependent variable changes when the associated explanatory variable changes, holding the other explanatory variables constant.⁵⁰ Again, suppose β_1 , the coefficient associated with COST, is equal to 0.90. In a multiple regression context this implies that the customer's price would be higher by \$0.90—holding DEMAND and PERIOD constant. Further assume that β_2 equals \$0.10, so that

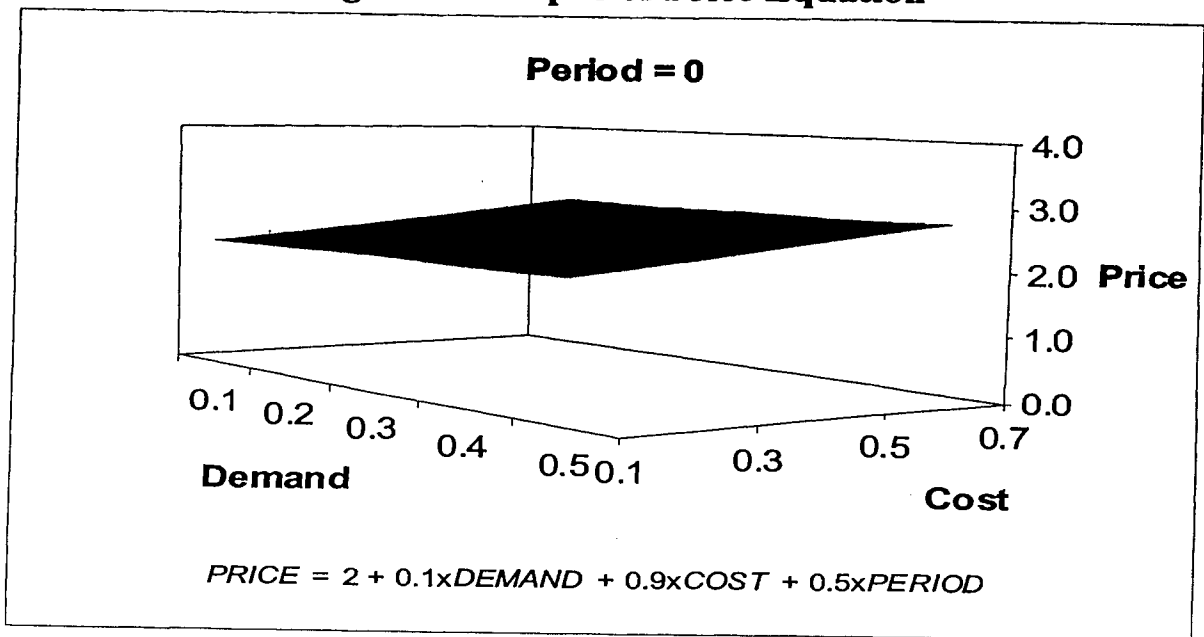
asserting invalid patents against its existing competitors. If before the resolution of the patent cases customers reacted to the pending litigation by switching their purchases to the monopolist (or failing to switch away from the monopolist), the damages to the competitors could in part be the profits on the sales they lost as a result. Given sufficiently reliable data, an econometric analysis of the amount of switching may be relevant both to causation issues and to the quantification of damages. Also note that a regression model used in practice often will have more than three explanatory variables. A relatively simple model is used here for expository purposes.

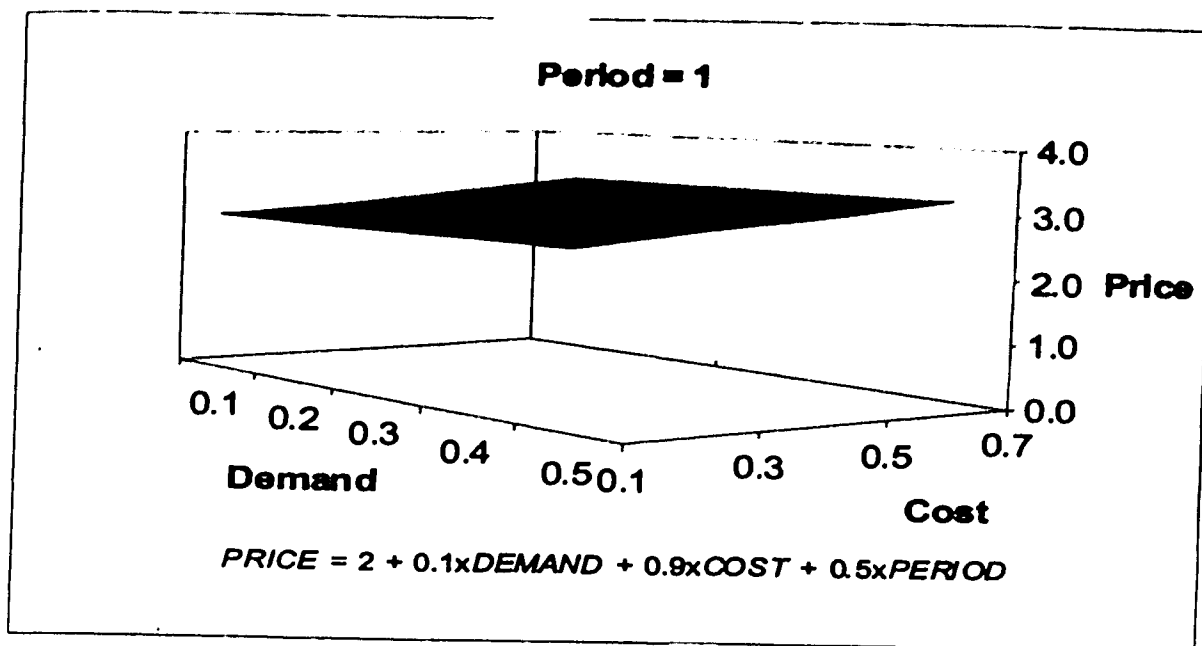
48. See Chapter 7 for an overview of the legal and economic issues involved in estimating damages associated with anticompetitive price increases or overcharges. As discussed below, the use of a single "dummy" variable for the conspiracy period may not be appropriate. See part E of this chapter.
49. See MADDALA, *supra* note 14, at 128-34.
50. See WOOLDRIDGE, *supra* note 3, at 14-16.

increasing DEMAND by 1 unit would increase price by \$0.10, holding COST and PERIOD constant. PERIOD takes on the value of zero in the before period and one in the during period. Similar to the other coefficients, β_3 measures the impact of PERIOD on price, holding COST and DEMAND constant. The example assumes this impact is \$0.50 for the during period.

Figure 2 graphically represents how price is related to DEMAND, COST, and PERIOD in three dimensions using the parameters above. The figure is in two parts, one for PERIOD = 0 (the before period) and one for PERIOD = 1 (the during period). As drawn in Figure 2, for each period, the relationship between price, demand, and cost forms a plane. For each point on the plane, increasing cost by \$1 increases the PRICE by \$0.90 (holding DEMAND and PERIOD constant) and PRICE increases by \$0.10 for each increase of one in DEMAND (holding COST and PERIOD constant). The effects of DEMAND and COST are the same whether PERIOD is equal to 0 or equal to 1. The effect of PERIOD is to raise the plane by \$0.50 as illustrated by the plane shifting upwards in the PERIOD 1 graph compared to PERIOD 0 graph.

Figure 2: Graphs of Price Equation





As a practical matter, the set of explanatory variables included in a regression model never accounts for all of the factors that affect the dependent variable.⁵¹ When there are many factors at work, there generally will be some factors that the econometrician can identify, measure, and include as explanatory variables, and some factors that cannot be identified and measured and thus cannot be included as explanatory variables. The relationship between the dependent variable, the explanatory variables, and the error term in the example would be expressed as follows:

$$PRICE = \beta_0 + \beta_1 COST + \beta_2 DEMAND + \beta_3 PERIOD + \varepsilon.$$

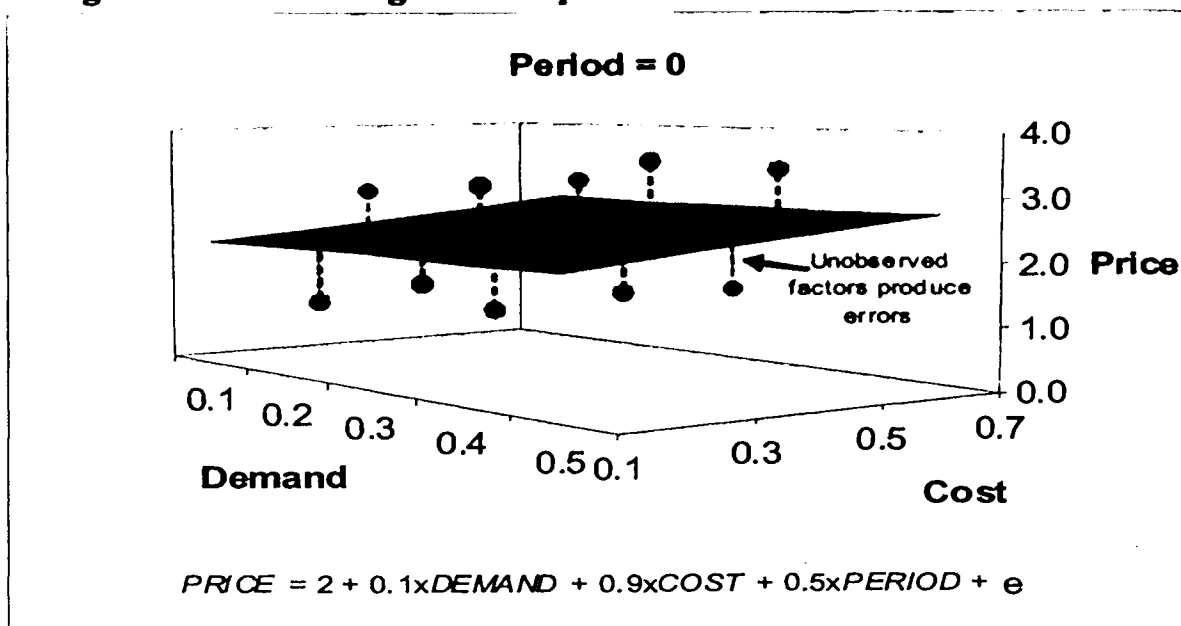
In this example, the unobserved factors that appear in the error term might include particular aspects about the supply and demand conditions facing the customer, such as the end uses to which the customer puts the product, the availability of substitutes for the product (which may differ across customers), the production level of the particular industry in which the customer participates (since the included demand variable

51. See KENNEDY, *supra* note 29, at 3.

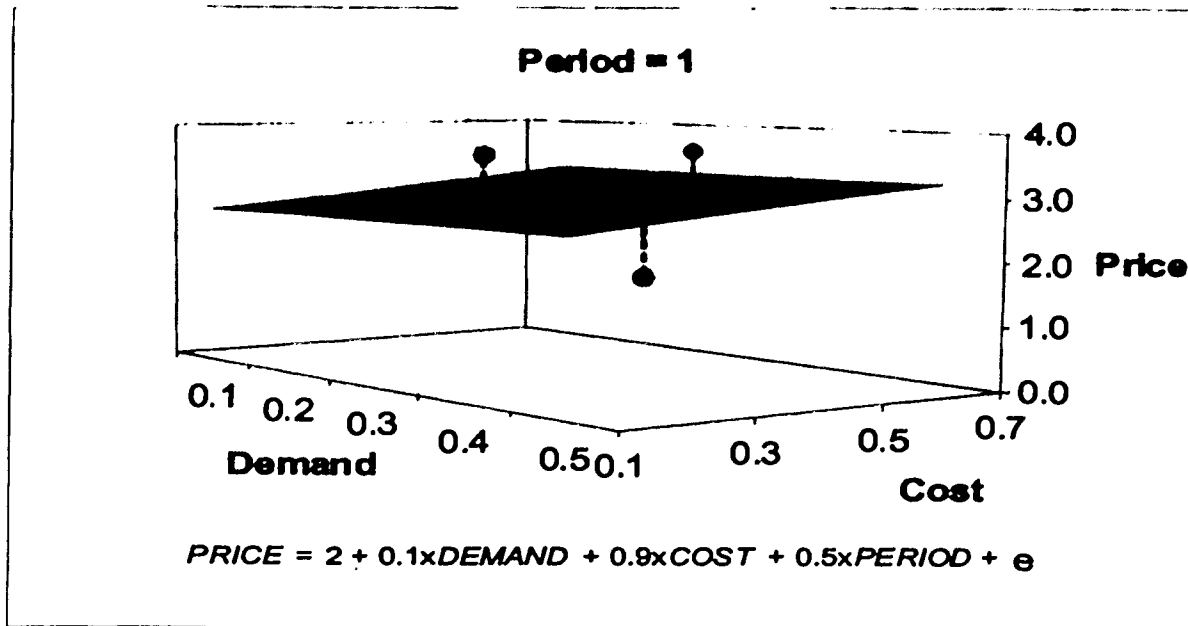
DEMAND may not be industry-specific), and the customer's negotiating skill if prices are individually negotiated.⁵²

Figure 3 shows the planes discussed above within the actual observations used in the regression, similar to the simple regression discussed above. The coefficients of the equation are estimated by minimizing the squared errors between the data points and the plane in order to get the "best fit" of the plane to the available data.

Figure 3: Estimating Price Equation From Actual Observations



52. See, e.g., John H. Johnson & Gregory K. Leonard, *Economics and the Rigorous Analysis of Class Certification in Antitrust Cases*, 3 J. COMP. L. & ECON. 341, 346-47 (2007).



3. Hypothesis Testing

Hypothesis testing is the use of statistics to assess whether the data are consistent with a specified hypothesis.⁵³ Typically the hypothesis will be a statement about a coefficient. For example, the hypothesis may be that the coefficient on the PERIOD variable is equal to zero. The hypothesis test will consider whether the estimate of the PERIOD coefficient obtained from the data is consistent with what would be expected if in fact the PERIOD coefficient were zero. As mentioned above, the estimated coefficient for PERIOD likely would not be exactly equal to zero even if the true underlying coefficient were zero. However, the estimated coefficient should not be "far" from zero if the hypothesis were true.

Testing a hypothesis about a coefficient requires a measure of the *statistical precision* of the estimate of the coefficient. If there is a great deal of statistical noise so that a coefficient estimate is highly imprecise, it would provide relatively little information about the value of the true underlying coefficient.⁵⁴ As an example, an opinion poll based on a very small sample of respondents generally would not be very precise and thus would not provide a very useful estimate of what percentage of the

53. See GREENE, *supra* note 16, at 147.

54. See KENNEDY, *supra* note 29, at 67-68.

overall population held the opinion. A larger sample of respondents would produce a more precise estimate, but getting information from the entire population is not necessary to get a reasonably accurate estimate.

To measure the precision of a statistical estimate, econometricians and statisticians typically use what is called a *standard error*.⁵⁵ The standard error is best defined by explaining how it is used to create a *95 percent confidence interval*. A 95 percent confidence interval is approximately equal to the range defined by the coefficient estimate plus and minus two times the standard error. One can be "95 percent confident" that the true underlying coefficient lies within the appropriately defined 95 percent confidence interval. Economists often use 95 percent confidence intervals, but they also sometimes use intervals defined by lower or higher levels of confidence, such as 90 percent or 99 percent.⁵⁶

When the coefficient estimate is relatively imprecise, the 95 percent confidence interval must be relatively wide to ensure that it includes the true value of the coefficient with 95 percent confidence. Conversely, when the coefficient estimate is relatively precise, the 95 percent confidence interval can be narrower.⁵⁷ Returning to the opinion poll example, pollsters often report the 95 percent confidence interval, e.g., "45 percent of respondents supported the proposal with a margin of error of +/- 5 percent." That is, the pollsters are 95 percent confident that the percentage of the population supporting the proposal is between 40 percent and 50 percent. The margin of error shrinks, all else equal, as the sample size of the poll increases and the precision of the poll increases accordingly. With a margin of error of +/-1 percent, the 95 percent confidence interval for the percentage of the population supporting the proposal would be narrower, from 44 percent to 46 percent.

Standard errors also can be used to conduct hypothesis tests regarding coefficients. Suppose the economist wishes to test the hypothesis that the coefficient on the PERIOD variable is equal to zero. To test this hypothesis, the economist would calculate the ratio of the coefficient estimate to its standard error, or " $\hat{\beta} / se(\hat{\beta})$ " where $\hat{\beta}$ is the

55. See WOOLDRIDGE, *supra* note 3, at 41.

56. See GREENE, *supra* note 16, at 146.

57. See KENNEDY, *supra* note 29, at 54.

coefficient estimate and $se(\hat{\beta})$ is the standard error of the coefficient estimate. This ratio is called a *t-statistic*.⁵⁸

If the hypothesis is correct and the true underlying coefficient is in fact zero, then the *t-statistic* should not be very far from zero. If the *t-statistic* turns out to be far from zero, it would cast doubt on the truth of the hypothesis. How do we determine whether the *t-statistic* is "far" from zero? We can calculate the probability that the *t-statistic* achieves a certain value, if the hypothesis were true. For example, if the hypothesis were true, there is about a 90 percent probability that the *t-statistic* will fall between 1.7 and -1.7 and about a 95 percent probability that the *t-statistic* will fall between 2 and -2. Thus, if the hypothesis were true, there would be only a 5 percent probability that the *t-statistic* we observe would be either greater than 2 or less than -2. Accordingly, if we observe a *t-statistic* greater than 2 or less than -2, the data would appear to be inconsistent with the hypothesis (because such an outcome is quite unlikely if the hypothesis were in fact true).

Indeed, if the absolute value of the *t-statistic* that the economist calculates exceeds two, then the hypothesis that the true underlying coefficient equals zero typically would be said to be *rejected* at the 5 percent significance level and the result typically would be termed *statistically significant*.⁵⁹ This result often is also expressed by saying that the coefficient is "statistically significantly different from zero (at the 5 percent level of significance)." The 5 percent level of significance (and the corresponding 95 percent confidence interval) is often used by economists and statisticians when conducting hypothesis tests, but other levels of significance, such as 1 percent or 10 percent, are also sometimes used.

As an example of these techniques of statistical inference, suppose that the coefficient estimate on the PERIOD variable in the price regression was 0.50, which would imply that prices were \$0.50 higher during the alleged conspiracy period as compared to outside that period, holding constant the variables COST and DEMAND (and assuming correct model specification). Suppose further that the standard error of the coefficient estimate on PERIOD is 0.20. In this case, the 95 percent confidence interval would be approximately \$0.10 to \$0.90—one can be 95 percent confident that the true underlying coefficient on PERIOD lies

58. See GREENE, *supra* note 16, at 249.

59. *Id.*

within this interval, assuming the validity and reliability of the econometric model and approach.

Similarly, the standard error can be used to test the hypothesis that the coefficient on the PERIOD value is zero, which implies that price was not higher during the alleged conspiracy period (holding constant COST and DEMAND). This test would be conducted by calculating the t-statistic: $0.50/0.20 = 2.50$. Since the calculated t-statistic of 2.50 is greater than two, the coefficient on PERIOD would be said to be statistically significantly different from zero at the 5 percent level of significance, and the hypothesis that the price was no higher during the alleged conspiracy period would be rejected.

4. Estimation of the Standard Errors

Correct statistical inference requires not only good estimates of the coefficients of the model, but also good estimates of the standard errors of these coefficient estimates.⁶⁰ For example, a t-test of whether a coefficient β is zero is conducted by forming the t-statistic " $\hat{\beta}/se(\hat{\beta})$," discussed in the last section. This t-statistic can be invalid and lead to incorrect statistical inference if the standard error of the coefficient estimate, $se(\hat{\beta})$, is itself inconsistently estimated.

Consistent estimation of the standard errors requires that the properties of the error term of the regression be properly taken into account. For example, standard errors frequently are estimated assuming the error term of the regression is *independently and identically distributed*. That is, the error for each observation reflecting the impact of unmeasured factors is assumed to be from the same distribution (or pattern) of possible errors, and each error is statistically independent of the others (they are not correlated to each other).⁶¹ If in fact the error terms are *correlated* with each other (i.e., not independent) or not identically distributed, then the resulting standard error estimates generally will be inconsistent.⁶²

Correlation among the errors of different observations can arise in various situations. For example, suppose the data sample is a *time series*, i.e., the data were generated by observing the variables (e.g., price, cost,

60. See WOOLDRIDGE, *supra* note 3, at 57.

61. *Id.* at 54-57.

62. *Id.*

and demand) at various points over time (e.g., on a monthly basis).⁶³ In such a case, the error in one month might well be related to the errors in adjacent months, since the unobserved economic factors that appear in the error term might themselves exhibit correlation over time. This correlation of errors over time is called *serial correlation*.⁶⁴

As another example, suppose the data sample is a *cross-section/time series*, or *panel data* set, where the variables are observed at various points of time separately for each of a number of units of observation such as individual customers.⁶⁵ Each customer's data are a time series. Therefore, the error terms for a given customer may exhibit serial correlation. In addition, each customer may have idiosyncratic factors that affect the price it paid, but are unobserved in the data. These factors would be present in all of the errors across time for that customer, which would be a further cause of correlation among the errors for a given customer. This effect is called an *unobserved individual-specific effect* (where the "individual" refers to the unit of observation, e.g., a customer).⁶⁶

Importantly, the correlation among errors need not be confined to errors that pertain to the same customer. For example, the error terms for all customers within the same time period also may be correlated. Unobserved economic factors may affect all customers' prices at a given point in time and therefore these common factors will appear in the errors of all of the customers in a given time period. Similarly, if these unobserved factors are themselves serially correlated, then the error for one customer in one month will be correlated with the error for another customer in another month. Therefore, there may be correlation among the errors both within and between units of observation in a panel data set.⁶⁷

There can be substantial consequences from estimating the standard errors for the coefficient estimates as if the errors were uncorrelated when they are in fact correlated. With positive correlation between the error terms, the incorrectly estimated standard errors generally will be

63. See GREENE, *supra* note 16, at 97.

64. *Id.* at 525.

65. *Id.* at 98.

66. See WOOLDRIDGE, *supra* note 3, at 248.

67. This problem is widely recognized in the econometrics literature. See Brent R. Moulton, *An Illustration of the Pitfall in Estimating the Effects of Aggregate Variables on Micro Units*, 72 REV. ECON. STAT. 334 (1990); Marianne Bertrand et al., *How Much Should We Trust Differences-in-Differences Estimates?*, 119 Q. J. OF ECON. 249 (2004).

biased downward, making the regression coefficients seem to be more precisely estimated than they really are. As a result, a statistical test on the coefficients may yield what appears to be a statistically significant result but is not.⁶⁸

To see why this is so, suppose that the estimate of the coefficient on a price-fixing conspiracy dummy variable is 0.15 and that the standard error, estimated by incorrectly ignoring correlation among the errors, is 0.05. The (incorrect) t-statistic would then be $0.15/0.05 = 3$ and the hypothesis that the alleged conspiracy had no effect on prices would be strongly rejected. But, the standard error is too low because it did not account for correlation in the error terms. Suppose that the standard error is re-estimated correctly to account for correlation in the error terms, and this correct standard error is 0.15. Then, the correctly calculated t-statistic is only $0.15/0.15 = 1$. Because the correct t-statistic is less than two, the hypothesis that the alleged conspiracy had no effect would not be rejected at conventional levels of statistical significance. If the standard error had not been corrected, the wrong inference would have been made.

Econometricians have a variety of methods for consistently estimating the standard errors when correlation among the errors exists. In a time series context (discussed in more detail below), various non-parametric procedures (procedures that do not impose any functional form on the correlation) have become widely used.⁶⁹ In a panel data context (also discussed in more detail below) these procedures may be used, and other methods have been proposed as well.⁷⁰ Some of these panel data procedures are easily implemented.⁷¹

68. *Id.*

69. See Donald W. Andrews, *Autocorrelation and Heteroskedasticity Consistent Covariance Matrix Estimation*, 59 *ECONOMETRICA* 817 (1991); Donald W. Andrews & J. Christopher Monahan, *An Improved Autocorrelation and Heteroskedasticity Consistent Covariance Matrix Estimator*, 60 *ECONOMETRICA* 953 (1992). The resulting standard errors are also consistent in the presence of *heteroskedasticity* (the variance of the error term differs across observations). Many statistical software packages implement the Newey-West procedure, which is one example of such a procedure to obtain autocorrelation and heteroskedasticity consistent standard errors. See Whitney Newey & Kenneth West, *A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix*, 55 *ECONOMETRICA* 703 (1987).

70. See Bertrand et al., *supra* note 67.

71. For example, Stata, a popular econometrics software package, includes a "cluster" option for calculating standard errors assuming unspecified

These procedures produce consistent estimates of the standard errors even when there is no correlation among the error terms. In other words, they work well in both situations. Thus, these procedures have become used more generally in practice.⁷² When there is good reason to suspect the existence of correlation among the errors, such procedures should be used to avoid making incorrect statistical inferences.⁷³

Finally, heteroskedasticity can also create problems in accurately measuring standard errors. Heteroskedasticity occurs when the variance of the error term varies across observations.⁷⁴ This condition is another violation of the independent and identically distributed error term assumption that can cause the traditional standard error calculation to be inconsistent. Again a well-known and widely used technique exists for calculating standard errors that are robust to heteroskedasticity (White standard errors).⁷⁵ This technique is also easily implemented in many econometric packages.⁷⁶

5. Choice of Explanatory Variables

As discussed above, the explanatory variables in an econometric model represent economic factors that influence the dependent variable.⁷⁷ An important question is which explanatory variables to include in the model. Answering this question should begin with economic theory combined with qualitative knowledge about the industry. For example, if the dependent variable is price, economic theory suggests that demand drivers, cost factors, and industry capacity, among other things, are potential explanatory variables.⁷⁸ Industry knowledge would suggest specific variables that would appropriately represent these factors. If a

within-group (cluster) correlation between the error terms. See 3 STATA PRESS, BASE REFERENCE MANUAL 81 (2007).

72. See GREENE, *supra* note 16, at 465 (“The . . . Newey-West estimator [is] becoming ubiquitous in the econometrics literature”).
73. If one is not sure, there are various tests one can run to test the hypothesis of no correlation in the error terms. See WOOLDRIDGE, *supra* note 3, at 130, 279, 420-449; GREENE, *supra* note 16, at 538-42.
74. See GREENE, *supra* note 16, at 499.
75. Hal White, *A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity*, 48 *ECONOMETRICA* 817, 838 (1980).
76. For example, Stata includes an option for calculating White standard errors. See STATA, *supra* note 71, at 81.
77. See part A of this chapter.
78. See Baker & Rubinfeld, *supra* note 2, at 391.

product is used as an input by downstream industries, the level of production in those industries might drive the demand for the product.⁷⁹ The prices of the inputs used to produce the product could be important to costs.

a. Too Many or Too Few Variables?

The number of explanatory variables suggested by economic theory and industry knowledge often will be large. Is it best to include all of the explanatory variables, or should one try to pare back the number of variables in order to have a simpler model? The downside to including extraneous explanatory variables in a regression is that the coefficients may be less precisely estimated. However, these estimates will still be unbiased.⁸⁰ Moreover, the effect on precision of having additional variables will often be small when the sample size is large.

Mistakenly excluding important explanatory variables in an attempt at simplicity, on the other hand, can result in an *omitted variable bias*. Omitted variable bias arises when important explanatory variables that have been omitted from the regression model are correlated with included explanatory variables. Because the omitted variables are in the error term, the result will be a correlation between the included explanatory variables and the error term.⁸¹ This misspecification will bias the resulting coefficient estimates, and make these estimates

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79. Care needs to be taken that the explanatory variables used in a least squares regression are *exogenous*, or uncorrelated with the error term, to the extent possible. See WOOLDRIDGE, *supra* note 3, at 50-51. For example, if the intermediate product in question represents a large share of the downstream industry's costs, the amount of downstream production may be affected by the price of the intermediate good. If the impact of the price of the intermediate good on the sales of the downstream product is substantial, then downstream production could be considered *endogenous* (correlated with the error term) rather than *exogenous*. See Baker & Rubinfeld, *supra* note 2, at n.17. Methods of detecting and dealing with endogeneity are discussed later in this chapter.
80. More specifically, ordinary least squares is still the best linear unbiased estimator as demonstrated by the Gauss-Markov Theorem, one of the more famous theorems in statistics. See GREENE, *supra* note 16, at 246. This means that not only is least squares unbiased, but also it is the most efficient (i.e., most precise) among linear unbiased estimators.
81. See WOOLDRIDGE, *supra* note 3, at 61-62.

unreliable for damage estimation. This bias does not diminish as sample size increases.⁸²

To see why, suppose that in the regression model discussed above, there is a second important demand driver variable, DEMAND2, that was omitted from the regression model. Suppose further that this variable was positively correlated with the alleged conspiracy period variable PERIOD. For example, DEMAND2 might be unusually high during the period of the alleged conspiracy, so DEMAND2 and PERIOD would be highly positively correlated. All else equal, when DEMAND2 was unusually high, PRICE should be unusually high. A regression model that included PERIOD, but did not include DEMAND2 would mistakenly attribute the effects of DEMAND2 (which was omitted from the model) to PERIOD, the variable that represents the alleged anticompetitive act. As a consequence, the regression would lead to the mistaken conclusion that the alleged conspiracy represented by PERIOD was the "cause" of the high prices, when in fact the cause was the unusually high values of DEMAND2 during the alleged conspiracy period. This bias will only occur, however, if DEMAND2 is empirically important for the dependent variable PRICE and correlated with PERIOD.

In general, one should be more concerned with avoiding bias than with improving precision. For these reasons, it is in general better to include more variables than fewer.⁸³ Nonetheless, no variable should be included unless there is an economic rationale for putting it in the model.

When there is concern that extraneous variables could greatly affect the precision with which coefficients can be estimated, a parsimonious model may have virtue.⁸⁴ There are several model selection criteria one can use to compare models containing different sets of explanatory variables.⁸⁵ Examples are the *Akaike Information Criterion* (AIC) and the *Schwartz Criterion* (SC).⁸⁶ These criteria reward increased explanatory power of the model, and penalize the addition of extraneous variables. When the penalty from the addition of a sufficiently weak explanatory variable exceeds the reward, the criteria will indicate that the variable should not be added. There are other sophisticated econometric methods that attempt explicitly to trade off bias against increased

82. *Id.*

83. See KENNEDY, *supra* note 29, at 95.

84. *Id.*

85. *Id.* at 206.

86. *Id.*

precision of the estimates.⁸⁷ However, these techniques should still only consider variables that economic theory and industry information identify as likely to be important and should not be used in a mechanical way.

b. Multicollinearity

Multicollinearity occurs when two or more of the explanatory variables are highly correlated with each other ("collinear").⁸⁸ When this happens, it may be difficult for the regression to distinguish which of the collinear variables is "important." In that case, the coefficient estimates on the collinear variables may be highly imprecise; that is, they may have large standard errors. It then becomes difficult to make statistical inferences about the true underlying coefficients. Dropping one or more of the collinear variables is sometimes argued to solve the multicollinearity "problem." This proposition is questionable and can lead to a significant bias in the estimates of the coefficients on the retained variables.⁸⁹ Dealing with multicollinearity requires recognizing several important points.

First, multicollinearity is a matter of degree. Explanatory variables are almost always correlated with each other to some extent; the question is, is the multicollinearity severe? Since multicollinearity is a characteristic of the sample of data rather than a characteristic of the underlying population, there is no rigorous statistical test for multicollinearity.⁹⁰ A number of indicators of multicollinearity have been developed, but many of these have shortcomings.⁹¹

87. Such techniques are often called "shrinkage" estimators. See KENNEDY, *supra* note 29, at 198. Because these techniques lead to biased estimators, there is no consensus among economists as to whether they should be used. *Id.* at 201.

88. Correlation coefficients measure how closely two data series are related to each other. When the two series form a perfectly straight line in a positive relationship ($y = \alpha x$), the correlation coefficient will be 1. When the two series form a perfectly straight line in a negative relationship ($y = -\alpha x$), the correlation coefficient will be -1. More generally, when the two series do not form a perfectly straight line but have random variations from a straight line ($y = (+/-)\alpha x + \epsilon$), the correlation coefficient will be between -1 and 1. See GREENE, *supra* note 16, at 75.

89. See KENNEDY, *supra* note 29, at 197.

90. DAVID A. BELSLEY ET AL., REGRESSION DIAGNOSTICS: IDENTIFYING INFLUENTIAL DATA AND SOURCES OF COLLINEARITY 95 (1980). Statistical tests allow one to make an inference about a population

Second, multicollinearity among a subset of the explanatory variables may affect the precision with which the coefficients on those explanatory variables are estimated, but not affect the precision with which the coefficients on other explanatory variables are estimated.⁹² Accordingly, even severe multicollinearity among a subset of explanatory variables may not affect the coefficient estimates that are of most interest in the analysis. In a price-fixing case, for example, interest typically focuses on the coefficient on the alleged conspiracy variable. Multicollinearity that affects the precision with which the demand variable coefficients can be estimated should be of little concern if it does not affect the precision with which the alleged conspiracy variable coefficient can be estimated. For this reason, a useful indicator for multicollinearity is that proposed by Belsley, Kuh, and Welsch.⁹³ This indicator focuses on identifying which explanatory variable coefficient estimates are potentially affected by the multicollinearity.

Third, if the precision of the coefficient estimate on the variable of primary interest (e.g., the alleged conspiracy variable) is not substantially affected by the multicollinearity, there is no reason to drop any variables that have a strong economic rationale for inclusion. As discussed above, dropping variables that are truly important could lead to omitted variable bias. A large change in the alleged conspiracy variable coefficient estimate after dropping collinear variables could indicate omitted variable bias, which would render the estimate of the alleged conspiracy variable coefficient unreliable. Therefore, in general the best course of action in that case would be to retain the collinear variables.⁹⁴

Now assume the variable of interest is affected by the multicollinearity. Would it now be valid to drop the other presumably economically important variable(s) with which the variable of interest is collinear? Generally no valid basis exists for choosing to retain the variable of interest as opposed to retaining the variable(s) with which it is collinear. The multicollinearity and the imprecision of estimation that results indicates that it is difficult to separate out the individual effects of

characteristic from the sample; but multicollinearity is a sample characteristic, not a population characteristic.

91. See KENNEDY, *supra* note 29, at 195.

92. See BELSLEY ET AL., *supra* note 90, at 91.

93. See *id.* Other indicators are "Klein's Rule of Thumb" and the "variance inflation factor." See GREENE, *supra* note 16, at 258.

94. See KENNEDY, *supra* note 29, at 197.

the collinear variables.⁹⁵ In that situation, there may be no "solution" other than gathering more data, which often may not be feasible. With severe collinearity between the variable of interest and other economically important explanatory variables, the coefficient on the variable of interest cannot be estimated precisely, and the regression model may therefore be unreliable for estimating damages. Dropping explanatory variables to reduce the multicollinearity without a sound economic reason does not fix the problem.⁹⁶

6. *R-Squared and Other "Goodness of Fit" Measures*

Most regression software packages generate so-called goodness of fit measures. These measures describe how well a statistical model fits a set of data. This discussion concentrates on the most common goodness of fit measure, *R-squared* (or R^2). *R-squared* is the percentage of the total variation in the dependent variable that is explained by the regression model. A higher R^2 means that the model explains more of the variation in the dependent variable. In the extreme, if the model were a perfect fit, the R^2 would attain its maximum possible value of 1.⁹⁷ If the model explained none of the variation in the dependent variable, the R^2 would attain its minimum possible value of 0.

A very small R^2 may raise concern about whether the model is missing important explanatory variables. In such instances, the model may not be capturing the dynamics of the market, or there may be important omitted variables that may bias the estimates.⁹⁸ However, one must not ascribe too much importance to R^2 , because a high R^2 (close to 1) is not required for a regression model to be reliable. Indeed, if one is

95. Jacques Dreze, Comment, *Nonspecialist Teaching of Econometrics: A Personal Comment and Personalistic Lament*, 2 *ECONOMETRIC REVIEWS* 291, 296 (1983), states this view quite well when he says that omitting a variable in this situation "amounts to elevating ignorance to arrogance."

96. *Id.*

97. KENNEDY, *supra* note 29, at 14. *R-squared* can be increased by simply adding variables to a regression, even if they are not statistically significant. To account for this problem, the *adjusted R-squared* is sometimes calculated. Adjusted *R-squared* increases when a new variable is added to the regression only if that new variable has a *t*-statistic greater than one. *Id.* at 94.

98. For a discussion, see Rubinfeld, *supra* note 20, at 179, 188, 216-17. However, even with a small R^2 , least squares produces unbiased estimates, as long as the conditional expectation of the error term given the included explanatory variables is zero.

interested in estimating the partial effects of the explanatory variables on the dependent variable, R^2 may not be particularly relevant if the omission of other exogenous variables does not create any bias in the estimation of coefficient for the variable of interest.

One must also take care when using R^2 to compare two regression models. Such a comparison can only be given any weight if the two regression models have the same dependent variable. Otherwise, the comparison is one of apples to oranges. For example, consider one model where the dependent variable is average list price and a second model where the dependent variable is the average transactions price (including discounts). Even if the first model explains a larger percentage of the variation in price, the first model is not necessarily "better" than the second model if the regression is intended to estimate the impact of an act on the prices customers pay. Moreover, even when the dependent variable is the same in two regressions, the model selection criteria discussed in the previous section are likely a better basis for comparison than R^2 .

R-squared is more relevant in a forecasting context, because a higher R^2 suggests that the model will provide forecasts with smaller confidence intervals. Nonetheless, when building a forecasting model, one must still be careful not to choose explanatory variables just to maximize the R^2 .⁹⁹ Making selections based on a higher R^2 can lead to "over-fitting" of the model to the data set on which the model was estimated.¹⁰⁰ While the model might fit those data very well, it may actually provide worse forecasts when applied to other situations because the model has not really captured the important underlying economic relationships. Again, when building a forecasting model, the model selection criteria discussed in the previous section should be used instead of R^2 .¹⁰¹

7. Misspecification Bias and Specification Tests

Regression analysis provides unbiased estimates of the regression model coefficients generally only if the regression model is correctly specified, so that the error term (the unexplained component of the dependent variable) is not correlated with the explanatory variables.¹⁰² If this condition does not hold, the regression model may be misspecified and may not produce reliable coefficient estimates. Thus, the model will

99. *Id.* at 219-20.

100. See GREENE, *supra* note 16, at 306.

101. *Id.* See also part D.5.a. of this chapter.

102. See WOOLDRIDGE, *supra* note 3, at 49-54.

not reliably measure the effect of the allegedly anticompetitive act on the variable of interest. Thus, from an economic and statistical point of view, it is important to test whether a regression model is misspecified in order to avoid drawing incorrect or unreliable conclusions.¹⁰³

Misspecification can arise from,¹⁰⁴ among other reasons, the imposition of incorrect restrictions on the regression coefficients, the assumption of an inappropriate functional form, or inclusion of endogenous variables as explanatory variables.¹⁰⁵ Endogenous variables are those that are caused, in whole or in part, by the dependent variable of the model.¹⁰⁶ For example, the quantity sold of a good is determined in part by its price. Thus, quantity is typically an endogenous variable in a regression model where price is the dependent variable.

In many circumstances, these problems can be remedied if detected and if the appropriate data are available.¹⁰⁷ Fortunately, there are well-known and generally accepted econometric tests for the presence of misspecification.¹⁰⁸ These tests often provide an objective and scientific means to determine whether a regression model is reliable.

For example, omitted variable bias can be tested by identifying and including in the regression model additional explanatory variables that economic reasoning and other market information suggest are likely to affect the dependent variable. If these additional explanatory variables turn out to be statistically significant, and the coefficient estimates on the previously included explanatory variables change substantially when the additional variables are added, then the regression model that omitted the additional explanatory variables likely is misspecified and its results are biased and unreliable.¹⁰⁹

Another potential misspecification is caused by imposing incorrect restrictions on the coefficients of the regression model—that is, by forcing the coefficients to take incorrect values. Forcing incorrect values on some coefficients can bias other coefficients.¹¹⁰ For example, suppose that the data contain information on two different customers (A and B),

103. See GREENE, *supra* note 16, at 701.

104. Another source of bias, the omission of important explanatory variables from the model, is discussed in part D.5.a. of this chapter.

105. See KENNEDY, *supra* note 29, ch. 6, 9, 10.

106. See WOOLDRIDGE, *supra* note 3, at 50-51.

107. *Id.* at 50, 51, 58.

108. *Id.* at 44; see also Jerry A. Hausman, *Specification Tests in Econometrics*, 46 *ECONOMETRICA* 1251 (1978).

109. See GREENE, *supra* note 16, at 337.

110. *Id.* at 281-82.

and that the regression model imposes the restriction that both customers have the same coefficients. Further assume that customer A has a perfect substitute to which it could turn in response to an anticompetitive price increase, while customer B has no substitute. In that case, even if there were a conspiracy, the alleged conspirators would be able to impose an anticompetitive price increase only on customer B. The existence of a substitute for customer A would prevent the alleged conspirators from imposing a supracompetitive price increase on customer A. If the coefficient on PERIOD were allowed to be different for the two customers, customer A would have a zero coefficient (since its price would not be higher during the alleged conspiracy period), while customer B would have a positive coefficient, say 10%, reflecting a 10% overcharge. However, a regression model that forced the PERIOD coefficient to be the same for the two customers likely would result in a positive coefficient estimate that was biased for both customers (too high for customer A and too low for customer B). For example, the estimate of the PERIOD coefficient might turn out to be 5%. Forcing the PERIOD coefficients to be the same for the two customers could lead to the incorrect inference that both customers were harmed by the alleged conspiracy, when in fact customer A was not harmed. This restriction may also significantly affect the accuracy of the estimate of total damages. For example, if customer B had purchased 100 units at \$1.10 and customer A purchased 10 units at \$1, the true total damages would be $(\$0.10 \times 100 + \$0) = \$10$. Yet, the results from the regression model in this example would lead to a total damages estimate of approximately $(\$0.05 \times 100 + \$0.05 \times 10) = \$5.50$, a substantial underestimate. The existence of bias resulting from the imposition of incorrect restrictions can be econometrically tested using standard procedures often used by economists.¹¹¹

8. *Structural Models Versus Reduced Form Models*

As discussed above, an expert must specify an econometric model that lays out the relationship between the dependent variable and the explanatory variables that represent the major economic factors potentially affecting the dependent variable. Econometric models can generally be divided into two broad types: *structural models* and *reduced form models*.¹¹²

111. *Id.* at 271-300.

112. See KENNEDY, *supra* note 29, at 171-72.

A structural model consists of equations that represent the basic economic relationships between variables.¹¹³ For example, the structural model for price and quantity in an industry might consist of a demand equation—representing how customer demand for the product is determined—and a pricing equation—representing how price for the product is determined. Price and quantity would be the endogenous variables, since they are determined by the other variables in the model.

Estimating structural models has certain advantages. For example, the parameters in the equations of a structural model generally have a specific economic meaning. For example, the coefficient on price in the demand equation would be related to the elasticity of demand. Nonetheless, structural equations can be difficult to estimate. Thus, analysts often instead estimate a “reduced form.”¹¹⁴ The reduced form is an algebraic rearrangement of an econometric model that has each endogenous variable on the left side of one equation and only exogenous explanatory variables on the right side. One can start with an explicitly specified structural model and then solve it to obtain the reduced form model, or one can directly specify the reduced form model. In this latter case, the endogenous variables price and quantity are expressed in terms of the exogenous variables and the error terms, and the coefficients on the exogenous variables in the reduced form are functions of (depend on) the structural parameters.¹¹⁵ One disadvantage of reduced forms is that their coefficients are generally a mixture of the structural parameters from the demand equation and the price equation and thus often do not have a direct economic interpretation.¹¹⁶

Should one use a structural model or a reduced form model? If the question of interest centers on the value of a structural parameter (e.g., the elasticity of demand), estimation of a structural model is generally a more straightforward way to proceed.¹¹⁷ However, if the question of interest concerns predicting an outcome variable on the basis of the exogenous variables, estimation of a reduced form model is generally a more straightforward way to proceed.¹¹⁸ For damages analysis in

113. *Id.*

114. *See Baker & Rubinfeld, supra* note 2, at 391-92.

115. *See KENNEDY, supra* note 29, at 171-72.

116. *See Baker & Rubinfeld, supra* note 2, at 392.

117. *Id.* at 405.

118. *Id.* One can predict the endogenous variables from a structural model by solving the model given the estimated structural parameters. However, this method is generally not as straightforward as estimating the reduced form model directly. *Id.* at 414-16.

antitrust cases, the question of interest often involves predicting the likely “but-for” value of an outcome variable, such as price.¹¹⁹ Accordingly, reduced form models will often be better suited to antitrust damages analyses. Even if one is going to estimate a reduced form, however, consideration of the basic elements of the appropriate structural model can be very useful in identifying the appropriate explanatory variables to include in the reduced form model since the reduced form model is obtained by solving the structural model for the endogenous variables in terms of the exogenous variables.¹²⁰

In some cases, the specified model may be a hybrid of a reduced form equation and a structural equation in that it includes endogenous variables as explanatory variables. If so, ordinary least squares (regular regression) can yield biased and inconsistent estimates.¹²¹ Instead, “instrumental variables” (IV) techniques will usually be more appropriate because they can still be consistent even when one of the explanatory variables is endogenous.¹²² While the absence of good instruments can cause problems for IV methods, the bias that results from ignoring the endogeneity problem and applying OLS could be substantial.¹²³

9. *Special Topics with Panel Data*

Data sets can be composed of single observations over a period of time (a *time series*) or of a number of observations in single time period (a *cross section*). In general, data sets consisting of both time series and cross sectional observations are called cross-section time series data—or panel data.¹²⁴

Panel data allow for richer econometric specifications than time series or cross-sectional data. Two often-used econometric specifications for panel data are *fixed effects* and *random effects*.¹²⁵ To understand the basics of these specifications, start with the usual regression model

119. *Id.* at 389.

120. *Id.* at 391.

121. See WOOLDRIDGE, *supra* note 3, at 83.

122. *Id.* ch. 5.

123. See Michael P. Murray, *Avoiding Invalid Instruments and Coping With Weak Instruments*, 20 J. ECON. PERSPECTIVES 111 (2006).

124. For a discussion of issues involved in using panel data, see part D.7. of this chapter.

125. See WOOLDRIDGE, *supra* note 3, at 284-91.

$$y_{it} = X_{it}\beta + \varepsilon_{it}$$

where i indexes the observations in a given period of time and t indexes different time periods. For example, if there were 100 transactions in each year (corresponding to a purchase by each of 100 customers) over five years, i would run from 1 to 100, t would run from 1 to 5, and there would be 500 total data points.

There may be reasons to believe that there are unobserved factors specific to each customer that affect the dependent variable such as price. For example, suppose some customers are better negotiators than others. Good negotiators are likely to receive better prices on each transaction than poor negotiators. Similarly, suppose some customers have better substitutes for the product in question than other customers. Then, the customers with good substitutes are likely to receive better prices on each transaction than customers with poor substitutes.

When these *unobserved individual-specific effects* are present, the error term of the regression can be modelled by breaking it into two parts.¹²⁶ The first part, representing the unobserved individual-specific effect, can be written as μ_i , indexed only by i . It is assumed to be fixed across all purchases for customer i . The second part of the error term can be written as η_{it} . It is indexed by i and t , indicating that it differs from one time period to the next for each customer. With this breakdown of the error term, the regression model can be rewritten as

$$y_{it} = X_{it}\beta + \mu_i + \eta_{it}$$

A regression of y_{it} on X_{it} can be run with a set of indicator (dummy) variables for each unit of observation (customer, in our example).¹²⁷ This is possible with panel data (but not a cross-section) because there are multiple observations for each customer over time. The dummy variable for customer i will control for the unobserved individual-specific effect for customer i . This is called a fixed effects

126. *Id.* at 251.

127. *Id.* at 265-79. Here, a dummy variable for customer i will equal one for that customer, and zero for all other customers. See MADDALA, *supra* note 14, § 8.2.

regression, where the fixed effect for customer i is equal to the regression coefficient on its unique dummy variable.¹²⁸

The use of fixed effects thus allows the econometrician to control for unobserved factors that do not vary across the observations for an entity, such as an individual in our example. Controlling for these factors is important because the unobserved individual-specific factors may be correlated with the explanatory variables. In that case, a regression of y_{it} on X_{it} without including the fixed effects would yield biased and unreliable results. Without controlling for the fixed effects, the regression in effect would suffer from omitted variable bias. Thus, a fixed effects regression has the benefit of providing reliable results, even if the unobserved individual-specific effect component of the error term is correlated with the explanatory variables.¹²⁹

For an example of how failing to control for fixed effects can bias a study's results, consider a study of the effects of education on individuals' earnings. An individual's motivation and inherent ability are difficult to measure fully with observable variables. In an earnings regression model, motivation and inherent ability therefore will appear in the error term as an individual-specific effect that will increase the individual's earnings over his or her lifetime. Yet, these same factors likely also increase the individual's education. Thus, education and the error term likely will be correlated. Without including fixed effects for the individuals in the regression, a regression of earnings on education (and other explanatory variables) may well produce biased results, since the coefficient on earnings would then also reflect the effects of motivation and inherent ability.¹³⁰

If the unobserved individual-specific effects are not correlated with the explanatory variables, fixed effects still yields reliable results, but a better estimator may be available. In particular, the random effects estimator may provide more statistically precise results than fixed effects. But, random effects also will be biased and unreliable if it turns out that the unobserved individual-specific effects are correlated with the explanatory variables. Before relying on random effects results, therefore, it is important to test the validity of the assumption of no correlation between the unobserved individual-specific effect and the explanatory variables.¹³¹

128. See WOOLDRIDGE, *supra* note 3, at 265-79.

129. *Id.* at 268.

130. *Id.* at 61.

131. See Hausman, *supra* note 108.

The *Hausman Specification Test* can be used for this purpose.¹³² Under this test, the fixed effects and random effects are compared. If the zero correlation assumption is valid, the two sets of results should be similar. If, on the other hand, the zero correlation assumption is invalid, then the random effects results (as well as least squares results) will be biased while the fixed effects results are still unbiased, so that the two sets of results should diverge.¹³³ In this case, the fixed effects model would yield more accurate results.

10. Special Topics with Time Series Data

In a time series context, model specification and estimation proceeds differently depending on whether the time series in question are *stationary* or *nonstationary*.¹³⁴ In effect, a nonstationary time series shows a significant trend upward or downward.¹³⁵ Such trends can create problems. For example, suppose a dependent variable is increasing at the same time an explanatory variable is increasing. A simple regression then could find a positive and significant relationship between the explanatory variable and the dependent variable, because both variables are independently trending upward at the same time, even though the two variables are not related. Moreover, the R^2 from an OLS regression has no meaning when data are not stationary. Many economic time series are thought to be nonstationary, such as a country's Gross Domestic Product or the price of a stock.¹³⁶

The properties of many econometric estimators, such as least squares regression, are derived under the assumption that the data series defining the dependent and explanatory variables are stationary. When the series

132. *Id.*

133. *Id.*

134. See KENNEDY, *supra* note 29, ch. 19. These issues should be considered if one has either a single time series, or panel data with reasonably long time series for each unit of observation. *Id.* at 287.

135. Technically, a stationary series has a finite variance, which means that in the future the series will not stray too far from its current values. See GREENE, *supra* note 16, at 752. A nonstationary series, however, exhibits infinite variance, which means that in the future the series will get infinitely far from its current value with positive probability. *Id.* at 776-81.

136. *Id.*

are nonstationary, a different set of econometric techniques should be employed.¹³⁷

a. Tests for Stationarity

A number of different methods for testing for stationarity in a time series have been proposed.¹³⁸ These methods generally test for what econometricians call "unit roots."¹³⁹ These unit root tests are not perfect, and one should view the results of the tests within the context of economic theory and the dynamics of the market. For example, suppose the time series of interest are price and marginal cost. In general, it would be economically implausible that one of these series is nonstationary while the other is stationary, because price usually is significantly influenced by marginal cost.¹⁴⁰ Now, suppose the results strongly reject the null hypothesis that marginal cost is nonstationary, but do not reject the null hypothesis that price is stationary. It would be reasonable to conclude from the results of the two tests, plus the implication of economic theory (suggesting both series must be either stationary or nonstationary), that both price and marginal cost are stationary in this case.¹⁴¹ If so, then standard regression analysis can be used.

137. *Id.*

138. For a brief introduction to stationarity tests, see MADDALA, *supra* note 14, ch. 14; see also GREENE, *supra* note 16, at 781-85; D. Kwiatkowski et al., *Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root*, 54 J. ECONOMETRICS 159 (1992).

139. See GREENE, *supra* note 16, at 780.

140. See *id.* at 790-91. There are some situations in which price might be stationary while cost was nonstationary, or vice versa. For example, cost and a third explanatory variable that affects price could both be nonstationary, but cointegrated (discussed further below). In that situation, a more complex econometric procedure must be used.

141. Similar considerations apply when one has panel data with long time series for each unit of observation. In many circumstances, it would be implausible for the price of one customer to be nonstationary while the price of another customer is stationary. See J. Hlouskova & M. Wagner, *The Performance of Panel Unit Root and Stationarity Tests: Results From a Large Scale Simulation Study*, 25 ECONOMETRIC REVIEWS 85 (2006).

b. Modelling Market Dynamics with Stationary Time Series

Although the usual econometric techniques can be applied to stationary time series, particular issues may arise when attempting to model the dynamics of such series. The starting point for modelling the dynamics is studying the economics of the industry and examining why dynamic effects may arise in the first place. That is, why do the economic outcomes not adjust instantaneously to changes in economic conditions, which would make modelling only contemporaneous influences on the dependant variable inappropriate? There may be long-term contracts or other institutional features that slow the speed of adjustment, so understanding the economic context can be important in accurately modelling the dynamics.

Three types of dynamics may need to be modelled. First, dynamic effects may arise if the dependent variable is influenced by earlier values. For example, where the dependent variable is price, the price in the current period can be affected by prices in prior periods if there are costs to firms changing their prices. These costs would discourage frequent price changes, so price changes would not occur as often and would likely be delayed until there was an accumulation of reasons to change price. Modelling this type of dynamics is often done by including lags (i.e., prior observations) of the dependent variable as additional explanatory variables.¹⁴²

Second, dynamic effects may arise if the impact of a change in an explanatory variable plays out over two or more time periods. This can occur, for example, if long-term contracts allow for adjustments to price due to changes in costs, but only after a lag. Modelling these dynamics is often done by including not only the contemporaneous value of an explanatory variable, but also values of the variable from earlier time periods as additional explanatory variables.¹⁴³

Finally, dynamic effects may arise if the error term includes unobserved factors that are themselves correlated over time. This induces *serial correlation* in the error term. The dynamics in this case may be limited to the error term, and can be addressed by modelling the form of the serial correlation.¹⁴⁴

The economics of the industry can suggest that dynamics of one type or another are present, and the modelling should focus on that form of

142. See GREENE, *supra* note 16, ch. 17.

143. *Id.*

144. *Id.* ch. 13.

dynamics. As discussed above, if there is evidence that sales are made under contracts that adjust to cost changes with a lag, then lagged costs likely should be included as an explanatory variable for estimating the impact of cost on prices. In the case where there are reasons to believe dynamics of all three types may be present, it is important that the modelling of all three types be done jointly, rather than sequentially. There may be important interactions between the three that would be missed if the modelling was done sequentially.¹⁴⁵

There is also an important question about the number of lags that should be included in the model. For example, if the analysis involves monthly prices and costs, should costs from just the previous month be included as an additional explanatory variable, or should the regression include costs from several previous months? The choice of lag length can be based on econometric tests, as we describe below. For example, a starting point for the modelling exercise might be the following model:

$$P_t = \sum_{j=1}^P \alpha_j P_{t-j} + \sum_{j=0}^Q X_{t-j} \beta_{t-j} + \varepsilon_t$$

$$\varepsilon_t = \sum_{j=1}^R \rho_j \varepsilon_{t-j} + u_t$$

where P, Q, and R represent initial choices of lag length for the lagged dependent variable, the lagged exogenous variables, and the serial correlation in the error term.¹⁴⁶ P, Q, and R should be chosen to be outer limits on the lag lengths. The optimal lag lengths then can be chosen by estimating the model using different combinations of the three lag lengths, and using model selection criteria such as the Schwartz criteria to choose between these models.¹⁴⁷

It is important to recognize that the specification above, which includes lags of the dependent variable as additional explanatory variables, potentially complicates estimation of the model. In particular, it creates a correlation between the explanatory variables and the error term if the error term itself exhibits serial correlation.¹⁴⁸ Under this

145. *Id.* ch. 17.

146. The error term could also be specified to have *moving average* components. For simplicity of presentation, we focus on autoregressive components only.

147. See GREENE, *supra* note 16, at 717.

148. *Id.* at 534-35.

condition, least squares estimation of the model can lead to biased and inconsistent estimates. If it appears that least squares estimation in this type of specification could lead to biased and inconsistent estimates, other econometric methods should be used (as discussed below).

It is also important to recognize that, as long as the rest of the model is properly specified, the appropriate estimation method will generally be unbiased and consistent even if the serial correlation in the error term is left unmodeled.¹⁴⁹ As discussed above, however, it will still be necessary to account for the serial correlation when estimating the standard errors to ensure correct statistical inference, and this can be done using the techniques described above.

c. Modelling Dynamics in Nonstationary Time Series

If the unit root stationary tests indicate that the series are nonstationary, a different set of econometric techniques in general should be used to estimate the relationships between the series.¹⁵⁰ The simplest approach is to take the observed value of a variable in each time period observation "t" and subtract from it the value of the same variable in the previous time period "t-1." Doing this calculation for each observation creates a new time series data set of the changes in the variable from one period to the next, and this transformation is typically done for all of the variables involved in the regression.¹⁵¹ This *first differences* approach frequently removes the nonstationarity from the data, although the transformed data should be tested to ensure that is the case.¹⁵² The approach, however, will also often remove some information about the relationship between the dependent variable of interest and the exogenous variables. Thus, goodness of fit measures such as R^2 and the measured significance of the statistical tests usually will be lower than regressions based on the untransformed data.¹⁵³

149. *Id.* at 533.

150. *See* MADDALA, *supra* note 14, § 6.10 & ch. 14.

151. *See* GREENE, *supra* note 16, at 790. Depending on the economics of the market and the nature of the alleged anticompetitive behavior, a dummy variable may not be transformed this way.

152. *Id.*

153. *See* GEORGE C.S. WANG & CHAMAN L. JAIN, REGRESSION ANALYSIS: MODELING & FORECASTING 88 (2003).

More sophisticated econometric techniques can be applied if the time series in question are nonstationary, but are *cointegrated*.¹⁵⁴ Two (or more) nonstationary series are said to be cointegrated if a linear combination of the different series is itself stationary.¹⁵⁵ For example, consider two time series. If one series plus a constant multiplied by the second series is stationary, the two series are cointegrated. Econometric methods exist for testing whether two or more series are cointegrated.¹⁵⁶

Cointegration can arise if economic forces prevent the nonstationary series from getting too far from each other, if they have a long-run relationship. For example, again consider a price series and a marginal cost series. While price and cost may each be nonstationary and trend upward, the economic forces at play would tend to keep the gap between price and marginal cost (i.e., the margin) from becoming too large or too small. If the margin grew too large, entry would occur or customers would cut back their purchases and drive the margin back down. If the margin grew too small, exit would occur or customers would increase their purchases and the margin would increase again. Thus, the margin can be stationary even though price and cost are themselves nonstationary.

If the tests for cointegration indicate that the series are cointegrated, then the relationship between the series can be specified and estimated as a *vector error correction model* (VECM).¹⁵⁷ A VECM models both the cointegrating equations (the long-run relationships between the nonstationary variables) and also the short-run dynamics of the residuals of the cointegrating equations.¹⁵⁸

154. If two series are nonstationary, but not cointegrated, regression techniques can still be applied. However, the estimated coefficients will have nonstandard sampling distributions. See Peter C.B. Phillips & Steven N. Durlauf, *Multiple Time Series Regression With Integrated Processes*, 53 REV. ECON. STUD. 473 (1986).

155. See GREENE, *supra* note 16, at 790.

156. *Id.* at 794-96. One of the most widely used of these tests is presented in Soren Johansen, *Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregression Models*, 59 ECONOMETRICA 1551 (1991).

157. See GREENE, *supra* note 16, at 793-94.

158. *Id.*

d. Considerations in Building a Forecasting Model

Time series models may be used to make forecasts. For example, as discussed below, a time series model may be estimated using data from a competitive period, and then used to forecast but-for prices during a conspiracy period.¹⁵⁹ Such a forecasting model obviously will be useful only to the extent that it provides reliable forecasts. For example, the start and end of a conspiracy period may be characterized by sharp "turning points" in a price series. Accordingly, it is important that the forecasting model be able to predict turning points that occur for nonconspiracy reasons. Otherwise, turning points due to nonconspiracy reasons might be incorrectly attributed to the conspiracy.¹⁶⁰

How does one tell if the forecasting model is reliable? Looking at R^2 is generally not a good way to assess a forecasting model. Models with lagged dependent variables or serially correlated errors often appear to track the historical data well and achieve a very high R^2 . However, as discussed previously, such models may suffer from a form of "overfitting."¹⁶¹ That is, while they fit the historical data well, they fail to capture the true economic structure of the dynamic pricing process. The good fit may come from the lagged dependent variables, not from the other explanatory variables that cause changes in the direction of the dependent variable. Thus, when such models are used to forecast prices during an alleged conspiracy period, they do a poor job forecasting turning points in the dependent variable.

One way to test a forecasting model is to see how well it predicts for some time period before the alleged anticompetitive act, assuming adequate data are available.¹⁶² To implement such a test, one would divide the data from before the alleged anticompetitive act into two parts: one part is used to estimate the forecasting model and the other part is used to assess the accuracy of the forecasts of the model. It is particularly useful to exclude from the part of the data used to estimate the model observations that contain a turning point (e.g., a substantial change in prices). If the forecasting model is reliable, then it should produce a reasonably accurate forecast for the part of the data not used in estimation. In particular, it should be able to predict any turning points. If the model does a poor job forecasting in this context, then it is unlikely

159. See part E.1. of this chapter.

160. See Baker & Rubinfeld, *supra* note 2, at 396.

161. See part D.6. of this chapter.

162. See KENNEDY, *supra* note 29, at 102.

to produce a reliable estimate of but-for prices during the period of the anticompetitive act.

E. Before-During Approach

The before-during approach identifies the effect of the alleged conduct by using data from a period before the alleged conduct in combination with data from the period when the alleged conduct occurred.¹⁶³ Comparing the values of the dependent variable in the before period to the values it took on in the during period may serve to identify the effect of the alleged conduct.¹⁶⁴ In making this comparison, however, it is critical to account for any significant differences in other economic factors between the before and during periods.¹⁶⁵

1. "Dummy Variable" Models Versus "Prediction" Models

For simplicity, assume that the best explanatory variable available for capturing the potentially anticompetitive effects is a dummy variable representing the time period when the alleged anticompetitive acts took place.¹⁶⁶ This variable will be 1 during that period and 0 at other times.

163. See van Dijk & Verboven, *supra* note 1, at 2335-36.

164. *Id.* The before period may be considered the control period and the during period the treatment period. See the discussion in part A of this chapter.

165. See van Dijk & Verboven, *supra* note 1, at 2335-36.

166. Depending on the nature of the antitrust allegations, the theory of causation, and specific facts of the case, there can be variables other than a dummy that may better measure the anticompetitive behavior in estimating damages. *Id.* Assume, for example, prices can only increase when existing customer contracts expire over time. The impact of the alleged conspiracy would not increase prices to all customers immediately, so average price increases related to the alleged conspiracy would be expected to trend upward over time. There could be several ways to model this effect other than using a simple dummy variable for the alleged conspiracy period, depending on the available data. *Id.* Under certain circumstances, one simple method to create a variable to approximate the conspiracy might be a "time trend" beginning at the period of the alleged conspiracy, with the value "1" in the first observation (e.g., a month) of the conspiracy period, "2" in the second, "3" in the third, etc. If average prices were the appropriate dependent variable, then the coefficient on this trend variable would be interpreted as the average monthly price increase due to the conspiracy. The

Specifically focusing on a reduced form model, one can account for the differences, if any, between the way price was determined in the during period (for example, with alleged price fixing) and the way price was determined in the before period (without price fixing). Suppose that the reduced form equation for price in the before period is

$$P_t^B = Z_t^B \Pi^B + v_t^B$$

where Z_t^B is a full set of exogenous explanatory variables (identified by any relevant economic analysis), Π^B are the estimated coefficients capturing the partial impact of these variables on the price P_t^B , and v_t^B is the error term. Suppose that the price-fixing conspirators set price by adding a fixed amount " Δ " to the price that would have prevailed otherwise. The "overcharge" resulting from the price-fixing conspiracy would then simply be Δ . In that case, the reduced form equation for price in the after period would be

$$P_t^D = Z_t^D \Pi^B + \Delta + v_t^D$$

The coefficients on the exogenous variables in the during period, Π^B , are the same as their coefficients on the exogenous variables in the before period. Holding constant the values of the exogenous variables and the error term, prices in the during period are simply increased by the amount Δ .

The equations for the before period and the during period can be combined to obtain a single equation that applies to both periods. This equation is

$$P_t = Z_t \Pi^B + \Delta A_t + v_t$$

overcharge in any month during the conspiracy would be the corresponding time trend value multiplied by that coefficient. However, care must be used with this and any approach for creating a variable that measures the impact of an alleged anticompetitive act. For example, at some point all contracts will have been renegotiated, so it would be inappropriate to continue to predict price increases beyond a certain number of observations.

where A_t is a dummy or indicator variable that takes the value of 1 if the time period t is in the during period and the value of 0 if the time period t is in the before period. Because of the crucial role played by the dummy variable, this type of model is often called a *dummy variable model*.¹⁶⁷

This model assumes that the coefficients on the predetermined variables are the same in the before and during periods. This assumption may not always be valid. For example, price-fixing conspirators may react differently to changes in supply and demand factors than they would have had they not been conspiring.

Thus, it may be important to test whether prices relate differently to the supply and demand factors in the during period than they did in the before period. In more technical terms, it can be important to test whether there has been a significant shift in the relationship between the exogenous variables and the dependent variable of interest.¹⁶⁸ Statistical tests of whether the additional restrictions inherent in the dummy variable model are appropriate can be performed by including *interaction* terms in the equation.¹⁶⁹ Interaction terms are variables calculated by multiplying other exogenous variables by the dummy variable.¹⁷⁰ Specifically, the reduced form price equation with interaction terms can be written

$$P_t = Z_t \Pi^B + \Delta A_t + A_t Z_t (\Pi^D - \Pi^B) + v_t$$

where A_t again is the dummy variable and $A_t Z_t$ is an interaction term. This equation will be equivalent to the reduced form equation of the dummy variable model if $\Pi^D = \Pi^B$, that is if the coefficients on the interaction terms will be zero. Thus, the appropriateness of the dummy variable model can be econometrically tested by testing the hypothesis $\Pi^D = \Pi^B$. If this hypothesis is rejected, this test suggests that the underlying assumption of the dummy variable model that the coefficients of the exogeneous variables in the before and during periods are the same is inconsistent with the data.

167. See Daniel L. Rubinfeld, *Econometrics in the Courtroom*, 85 COLUM. L. REV. 1048; James F. Nieberding, *Estimating Overcharges in Antitrust Cases Using a Reduced-Form Approach: Methods and Issues*, J. APPLIED ECON., Nov. 2006, at 361, 367-69.

168. See GREENE, *supra* note 16, at 287-93.

169. *Id.*

170. *Id.* at 326.

If the dummy variable model is found to be inappropriate, an alternative is a *prediction model*.¹⁷¹ The first step in implementing a prediction model is to estimate the reduced form equation on the data from the before period. Then, this equation is used to predict the prices that would have prevailed in the during period, given the values of the exogenous variables in the during period. The "predicted" price for time t of the during period, \hat{P}_t^D , can therefore be written as

$$\hat{P}_t^D = Z_t^D \Pi^B$$

where Z_t^D are the values of the exogenous variables in the during period and Π^B are the coefficients for these variables estimated based only on data from the before period. As can be seen from this equation, the prediction model assumes that the coefficients on the exogenous variables would have been the same in the during period in the but-for world as they were in the before period in the actual world. But, under certain conditions, this assumption may be invalid. For example, assume there was a change in the relationship between the exogenous variables and the variable of interest (e.g., price) in the during period that had nothing to do with the alleged anticompetitive conduct (e.g., entry or exit of firms from the market) and the prediction model does not take that change into account. Under these conditions, estimates using data solely from the before period may not accurately predict pricing but for an alleged conspiracy or other anticompetitive act.¹⁷² Moreover, as discussed above, a prediction model should be tested where possible to check whether it accurately predicts turning points in the data.¹⁷³ However, unlike the dummy variable model, the prediction model does not assume that the coefficients for the exogenous variables are the same across the before and during periods in the actual world.

171. See MADDALA, *supra* note 14, § 4.7; Nieberding, *supra* note 167, at 367-69.

172. In this way, the prediction model may suffer from some of the same limitations related to the dummy variable model discussed above.

173. A dummy variable model can suffer from a similar problem of failing to fit turning points well, and thus incorrectly attributing to the conduct turning points that may have nothing to do with the alleged conduct. See WOOLDRIDGE, *supra* note 3, at 61-62.

In addition, one also could fit the model to the during period and use the model to predict back ("backcast") into the before period. To the extent that the backcast is above the actual prices in the before period, that would suggest the price setting process in the during period produced higher prices than the price setting process in the before period. This finding would be consistent with the proposition that the alleged anticompetitive behavior in the during period increased prices, and the difference between the actual and predicted prices in the before period could form the basis for calculating an overcharge and damages.

2. Before-During-After Models

The previous section discussed before-during models that assume there is a single unaffected period (the before period). Sometimes, data might exist on several unaffected periods. For example, data may exist not only for the before period, but also for an after period. With more than one unaffected period, the question arises as to whether one should combine the two unaffected periods and, if not, then which of the two unaffected periods should be compared to the affected period.¹⁷⁴

The before and after periods should be combined, if appropriate, since that will result in a larger sample and more precise estimation.¹⁷⁵ It is appropriate to combine them if the parameters of the reduced form equation are the same across the two periods.¹⁷⁶ Suppose the following are the reduced form equations for price, one for the before period and one for the after period:

$$P_t^B = X_t^B \Pi^B + u_t^B$$

$$P_t^A = X_t^A \Pi^A + u_t^A$$

If the relationship between price and the exogenous variables in the two periods is the same, then the coefficients are the same for each period ($\Pi^B = \Pi^A$). In that case, it would be appropriate to combine the two periods together and estimate a single model that applied to both.

174. See van Dijk & Verboven, *supra* note 1, at 2336. Of course, it is important to ensure that the supposedly unaffected periods really are unaffected.

175. See MADDALA, *supra* note 14, at 69, 270; A.H. STUDENMUND, USING ECONOMETRICS: A PRACTICAL GUIDE 68 (4th ed. 2001).

176. See TAKESHI AMEMIYA, ADVANCED ECONOMETRICS 399-402 (1985).

The hypothesis $\Pi^B = \Pi^A$ can be econometrically tested by estimating separate models for each period and then comparing the respective coefficients to see if they are the same, apart from statistical variation.¹⁷⁷

The difficult decision arises when this test rejects combining the two periods into a single model and one must choose which unaffected period is the better basis of comparison for the affected period, assuming both are truly unaffected. Before addressing how to decide which unaffected period to use, first consider why two unaffected periods might exhibit different relationships between the dependent variable and the explanatory variables. There are economic models of competition that suggest that periods of more intense competition ("price wars") will alternate with periods of less intense competition.¹⁷⁸ These models suggest that the relationship between price and the explanatory variables may change over time with the intensity of competition. In this case, a before period that was dominated by more intense competition might look different than an after period that was dominated by less intense competition, even though both periods were unaffected by the anticompetitive conduct at issue.¹⁷⁹ Alternatively, there may have been substantial differences in market conditions between the two periods (e.g., there may have been substantial new entry into the market), which led to the differences in the model coefficients.

In the event that there are substantial differences between the before and after periods, one needs to determine how to select which period to use. One way to answer this question is to try to model explicitly the switches between competitive regimes, and then use this model to predict which competitive regimes would have prevailed during the affected period in the but-for world. There are econometric techniques called *switching models* that can be used to do this.¹⁸⁰ Although not commonly done, using a switching model of this sort may also provide a more realistic picture of competition in the during period. In contrast to the implicit assumption of the dummy variable model, a price-fixing conspiracy may be subject to breakdowns, where price would return to competitive levels (or even subcompetitive levels) for a period before the conspiracy price level was reestablished. Failure to model this possibility may lead to less precision in the estimate of the damages, or

177. See GREENE, *supra* note 16, at 287-93.

178. See, e.g., Jerry Green & Robert Porter, *Noncooperative Collusion Under Imperfect Price Information*, 52 *ECONOMETRICA* 87 (1984).

179. *Id.*

180. See AMEMIYA, *supra* note 176, at 399-402.

possibly even to an unreliable estimate of damages if explanatory variables are correlated with the onset of price wars because of omitted variable bias.¹⁸¹

An alternative approach for choosing between the before and after periods can involve using general market information and nonquantitative evidence from the case. For example, suppose that the after and during periods were characterized by strong demand for the product, while the before period was characterized by weak demand. These factors would suggest that the after period would provide a better basis of comparison for the affected period than would the before period. It is also possible that the effects of the anticompetitive conduct at issue last beyond the time when the conduct actually ended. If so, then the after period is not truly "unaffected" and the before period may be the superior basis for comparison.

Another approach, more conservative from the point of view of plaintiffs, is to predict price (absent the allegedly anticompetitive acts) for the affected period alternatively using the before and after periods, and then take the maximum of these two predictions. Both of these predictions are consistent with competitive behavior. Using the higher of the predicted prices but for the alleged anticompetitive acts compared to the actual prices will yield a lower estimate of damages, since damages in this instance are based on the difference between the actual and but-for prices.

3. Identifying Beginning and End Points of the Damages Period

The beginning and the end of the damages period alleged in many cases may not accurately reflect the actual beginning or end of the alleged unlawful conduct. For example, in price-fixing class action cases, the plaintiff attorneys often choose the beginning and end dates for the "class period" before discovery is undertaken. Moreover, the beginning or end of the effects of the alleged unlawful conduct may not coincide with the beginning or end of the conduct itself. The effects might start later, end earlier, or last longer than the conduct. Accordingly, experts should rely on the evidence developed in discovery, market facts, and the analysis of liability experts when determining the relevant starting and ending dates for calculating damages.

Accordingly, damages experts should be flexible when determining the appropriate beginning and end dates for damages, and econometric

181. See part D.5.a. of this chapter.

analysis may be able to assist in this respect. Recently, econometric methods have been developed that can be used to choose beginning and end dates based on market data.¹⁸² These methods involve estimating the point in time when an econometric model exhibits a *structural change*, or shift in the model parameters. In the case of alleged anticompetitive conduct that had an effect, a structural shift in the econometric model should have occurred when the effect of the conduct began and a second structural shift should have occurred when the effect of the conduct ended. Thus, for example, identifying when there were structural shifts in the regression model can provide useful evidence concerning the beginning and end of overcharges from a price-fixing conspiracy.¹⁸³

F. Benchmark and Difference-in-Differences Approaches to Damages

The benchmark approach to damages is based on the same basic principles and econometric tools as the before-during-after approach. However, in a benchmark analysis, the variable of interest (e.g., price) in the affected market is compared to that variable in an unaffected benchmark market.¹⁸⁴ Instead of using past prices in the affected market as a control, the benchmark approach uses prices in the unaffected market as a control. The benchmark market (or markets) can be a different area or a different end use market. The key is that the benchmarks (the control group) were unaffected by the alleged anticompetitive conduct. This approach may be particularly useful if there are inadequate data for the before period, so a before-during approach cannot reliably be performed.

One potentially significant problem with the benchmark approach is that differences in economic conditions between benchmark and affected markets make it likely that prices in those markets would not be identical, even if there were no anticompetitive behavior.¹⁸⁵ Comparing variable profit margins between the market of interest and the benchmark

182. Jushan Bai & Pierre Perron, *Estimating and Testing Linear Models With Multiple Structural Changes*, 66 *ECONOMETRICA* 47 (1998). See Dennis Carlton & Gregory K. Leonard, *Correcting the Bias When Damage Periods Are Chosen to Coincide With Price Declines*, 2004 *COLUM. BUS. L. REV.* 304 (2004), for a discussion of the application of these methods to price-fixing cases.

183. See Carlton & Leonard, *supra* note 182.

184. See van Dijk & Verboven, *supra* note 1, at 2336.

185. *Id.*

can control for many cost- and demand-based differences in prices, although there are recognized problems in calculating and comparing economically meaningful profit margins.¹⁸⁶

The difference-in-differences approach also can be used to account for at least some of the persistent unobserved differences between the benchmark market and the market of interest. This approach requires data for the market of interest and the benchmark that cover both a period of time that is unaffected by the alleged anticompetitive act and the period of time when the market of interest allegedly was affected. Thus, the difference-in-differences approach combines the before-during and benchmark approaches.¹⁸⁷

The difference-in-differences approach can be implemented using a regression that takes into account the various factors that differ between markets and time periods. This approach in general would use the equation

$$P_{it} = \mu_i + \delta_t + \theta D_{it} + \varepsilon_{it},$$

where i indexes markets (affected or unaffected), t indexes time, μ_i is a market-specific unobservable effect that is constant over time, δ_t is a time-specific effect that is constant over markets, D_{it} is a dummy variable equal to one if the market is the affected market and the time period t is during the affected period, and θ is the partial effect of the alleged anticompetitive conduct.

The difference-in-differences method can be expanded by using the following regression equation to control for other economic factors that might have changed over time.

$$P_{it} = X_{it}\Pi + \mu_i + \delta_t + \theta D_{it} + \varepsilon_{it},$$

186. See, e.g., Franklin Fisher, *On the Misuse of the Profits-Sales Ratio to Infer Monopoly Power*, 18 RAND J. ECON. 384-96 (1987). However, if the benchmark and the market of interest can be shown (1) to have similar supply and demand characteristics and (2) to have similar accounting methodologies, then one may be able to estimate the difference in those margins and from that calculate whether prices are higher in the market of interest—even if data are only available in the during period.

187. See WOOLDRIDGE, *supra* note 3, at 130.

where X_{it} are exogenous variables specific to market i in period t .¹⁸⁸

G. Summary of Key Points in Creating and Evaluating Econometric Models of Damages

As can be surmised from the previous sections, a number of considerations should be taken into account when building or evaluating an econometric model used to estimate damages.¹⁸⁹ Model specification and estimation involve many choices: explanatory variables, functional form, dynamic specification in a time series, estimation technique, and which of the basic approaches to damages to use. In many cases, statistical tests can help guide these choices. However, the starting point for any econometric model is economic theory and knowledge of the industry, and being sure that there are sufficiently reliable data to perform an econometric analysis.

Economic theory provides guidance as to the general types of explanatory variables that should be included in the model. For example, for an econometric model of price, economic theory suggests that variables representing cost, demand, and competitive factors would be important types of explanatory variables.¹⁹⁰ Knowledge of the industry helps determine with more specificity which variables to include. For example, if the industry in question were plastics, the price of petroleum (an important input to plastics) likely would be a relevant cost variable and production levels in downstream industries that use plastics likely would be relevant demand variables.

One must be able to obtain enough sufficiently reliable data to implement any economically sound econometric model. Econometric analysis based on very few data points in general will be much less reliable than econometric analysis based on a larger data set. There may also be issues with how accurate the data are, and whether they represent an unbiased sample of what has occurred in the market. Accordingly, the data need to be checked for accuracy and any potential selection bias. Finally, in many instances the data will need to be "cleaned" to some extent, removing obviously incorrect observations. For example, many damages analyses use data from a company's sales database. Care must

188. *Id.*

189. Much of the discussion here and elsewhere in this chapter can be applied to any econometric analysis. See, e.g., WOOLDRIDGE, *supra* note 3; GREENE, *supra* note 16.

190. See van Dijk & Verboven, *supra* note 1, at 2335.

be taken with such data to ensure that all significant discounts have been accounted for, and to weed out any obvious errors.¹⁹¹ In some circumstances, limitations on the availability or quality of data may make it impossible to conduct a meaningful econometric analysis.

Often economic theory and industry knowledge might suggest a large number of potential explanatory variables. While there is a benefit to parsimony (namely potentially increased precision of the estimators), there is typically a greater concern with omitting a potentially important explanatory variable that can induce omitted variable bias. Thus, generally speaking, one should exercise great care if potentially important variables are to be excluded from a model.¹⁹² In general, statistical tests or the AIC and SC criteria should be employed before dropping variables. It is rarely advisable to drop a theoretically important variable, especially if its coefficient is statistically significantly different from zero. Similarly, one generally should not drop a group of theoretically important variables when their coefficients are jointly statistically significantly different from zero (even if they are individually not statistically significantly different from zero).¹⁹³

The relationship between the dependent variables and the explanatory variables can be expressed in various functional forms of the underlying variables, and this is another important aspect of model specification.¹⁹⁴ For example, the dependent variable might be the unadjusted price of the product at issue and an exogenous variable could be variable costs. This model would provide an estimate of how much price would go up for a dollar increase in cost. However, there may be good theoretical or industry specific reasons to believe that the relationship of price to cost is better measured in percentage change terms for each variable. If so, then the functional form of the estimating equation may be better specified using the logarithms of price and cost as variables. Statistical tests can help determine which functional form is most appropriate for the situation at hand.¹⁹⁵

In a time series context, specification of the dynamics is important. Again, the particular specification might be guided by industry

191. For example, frequently these transactions data sets include credits for returned goods, etc. It is important to use net revenue and net sales to calculate a measure of the price actually paid.

192. See part D.5.a. of this chapter.

193. See KENNEDY, *supra* note 29, at 94.

194. See part D.1. of this chapter.

195. *Id.*

knowledge, and statistical tests, and the AIC and SC can help make choices about (for example) the lag lengths employed.¹⁹⁶

Care must be taken to specify correctly the structure of the error term in the estimation of the econometric model and to use estimates of the standard errors that are robust to deviations from the assumption of independent, identically distributed error terms. Only with correct standard errors will the resulting statistical inference be valid. The correct method for calculating standard errors can be determined based on the outcome of statistical tests, and in general such tests should be performed.¹⁹⁷

The decision of which of the basic damages models to use can be of great importance. A reduced form model typically will provide the most efficient and simplest model for damage estimation. However, the explanatory variables to be included in such a model need to reflect the important underlying supply and demand aspects of the market, and in some instances estimating the underlying structural model will be a superior approach.¹⁹⁸

The decision regarding which damages approach to employ also will depend on the available data and the nature of the market. Are there sufficient data to implement a before-during or before-during-after model? Is there a good benchmark? Can a difference-in-differences approach be used? In general, if reliable data are available from outside the period of that alleged anticompetitive act, then techniques that take advantage of those data will likely be the most reliable.¹⁹⁹

Deciding whether to use a dummy variable or prediction model can also be important. Is there evidence that the dummy variable technique is unreliable due to substantial changes in the model coefficients due to the alleged illegal act? Will the prediction model yield reliable results? Again, there are statistical tests that can help determine if one approach is clearly superior to another.²⁰⁰ Since neither approach likely is perfect, in the absence of strong statistical test results favoring one or the other, it may make sense to use both approaches and see if they yield similar results.

Once the econometric model is specified and estimated, another series of statistical tests and checks can be run on the reliability and

196. See part D.10. of this chapter.

197. See part D.4. of this chapter.

198. See part D.8. of this chapter.

199. See parts E and F of this chapter.

200. See part E.1. of this chapter.

robustness of the estimates.²⁰¹ The starting place is typically examining the sign and statistical significance of the coefficients. If, for example, the coefficient on cost in a price regression is negative and statistically significantly different from zero, this would cast doubt on the reliability of the model, since we would normally expect cost to have a positive effect on price. When performing this sort of check, however, one must take care that there is a clear interpretation to the coefficients; there may not be one in a reduced form model, for instance.²⁰²

When there is reason to question a model's specification, a variety of specification tests can and should be employed. For example, as discussed above, a model may impose potentially questionable restrictions on the coefficients, such as that the coefficients are the same during two time periods. Specifically, an econometric model of pricing at the customer level might assume that all customers have the same coefficients on the cost and demand variables. However, there may be reason to think that customers have different responses to cost and demand conditions, so there could be different coefficients on the explanatory variables across customers, including the variable measuring damages. In this case, it may be necessary to estimate the model for different groups of customers, or otherwise to employ a more complex model that takes these differences into account.²⁰³ Similarly, a time series model might assume that the coefficients are the same over the course of the entire time period, but there may be evidence that the structure of the model changed at some point in time. A Chow test (or a more sophisticated test for structural breaks) can be used to test whether such a change occurred.²⁰⁴ If so, the appropriate model needs to adequately control for such a change.

There may be reasons to question the crucial assumption that the error term of the regression is uncorrelated with the explanatory variables. This assumption often can be tested using a form of the Hausman Specification Test. For example, if we are concerned that an explanatory variable might be endogenous (which would bias OLS regression results), then the Hausman Specification Test comparing the

201. See part D.7. of this chapter.

202. See Baker & Rubinfeld, *supra* note 2, at 392.

203. See, e.g., Johnson & Leonard, *supra* note 52, at 351-52.

204. The Chow test is a statistical and econometric test of whether the coefficients in two linear regressions on different data sets are equal. See Gregory C. Chow, *Tests of Equality Between Sets of Coefficients in Two Linear Regressions*, 28 *ECONOMETRICA* 591 (1960).

OLS results to the IV results can be used to check for this problem.²⁰⁵ If such a problem is found to exist, then more complex econometric techniques can be used to correct for the bias.²⁰⁶

Finally, virtually any regression model eventually will fail one or more tests if enough tests and specifications are run, even if nothing is wrong with the model. However, failure of a test should be taken seriously, and a model should be rejected when it fails a test of a critical assumption, or if it fails a large number of the specification tests to which it is subjected.²⁰⁷

H. Case Study—The Application of the Before-During and Benchmark Approaches

Econometric techniques can be applied in a variety of ways to estimate the impact of the alleged anticompetitive conduct on certain economic outcomes, such as estimating lost sales or price changes due to the alleged conduct. Some of the econometric techniques and tests discussed above and their applications are illustrated using the before-during and benchmark approaches to estimate the price effects of alleged collusive behavior by analyzing actual data relating to Southern Powder River Basin (SPRB) coal.

1. Background

Virtually all coal produced in the United States is used for electric power generation.²⁰⁸ About one-third of the coal now comes from the SPRB, which contains vast amounts of low-sulfur coal available in low-cost surface mines. SPRB coal is relatively homogeneous, with two main types: 8400 Btu/lb. coal and 8800 Btu/lb. coal. SPRB coal mining began with the 8400 Btu coal, which was closer to the surface and easier to mine. During the 1970s, many coal-powered electric generation units were constructed to burn this coal. Virtually all of these units also can burn the higher-quality 8800 Btu coal. As sulfur emissions regulations became more stringent, older electric generation units were converted from burning 12,000 Btu bituminous coals to burning 8800 Btu SPRB

205. See Hausman, *supra* note 108.

206. See WOOLDRIDGE, *supra* note 3, ch. 5.

207. See KENNEDY, *supra* note 29, at 76-79.

208. U.S. ENERGY INFORMATION ADMINISTRATION, ANNUAL COAL REPORT 2007 9 (2009), available at <http://www.eia.doe.gov/cneaf/coal/page/acr/acr.pdf>.

coals. Because these plants already suffer capacity losses from burning lower Btu SPRB coals, they generally will not burn 8400 Btu coal.

During 1999 and 2000, the three leading SPRB coal producers conducted what economists may describe as “cheap talk.” Cheap talk refers to communications between oligopolists about future intentions. Such talk can facilitate understanding of each other’s goals and intentions, which can lead to collusive production and pricing.²⁰⁹ The statements were of the nature that they would reduce production “until prices improved.” The largest producer, Peabody Energy, closed a mine in 1999. The third largest producer—Arch Coal, Inc.—closed another mine in 2000, and several other planned expansions were cancelled. The result was a spot price increase in 2001. In the aftermath of the capacity reductions, prices settled about 30 percent above the level before the shut-downs. The DOJ opened an investigation in July 2001 but ultimately took no action against the SPRB producers.

2. Economic Model, Data, and Choice of Explanatory Variables

For the purpose of this case study, the simplest economic model hypothesizes that if the alleged collusion had an effect, then prices of SPRB coal during the damages period should be higher than before the damages period after controlling for other supply and demand influences. As will generally be the case, the most straightforward econometric models for estimating damages use a reduced form, rather than a structural form,²¹⁰ but the model still needs to include the major supply and demand influences on price that would be included in a structural model. Here, we assume a linear relationship between the relevant market forces and SPRB prices. A full analysis would investigate (and potentially test) whether the relationship of these factors to price should be specified differently, such as in percentage change terms.²¹¹

The price of SPRB coal used in the analysis is the spot price reported by the utilities that purchased the SPRB coal.²¹² The utilities reported the

209. See MICHAEL D. WHINSTON, LECTURES ON ANTITRUST ECONOMICS 20 (2006). Whether such behavior constitutes an unlawful price-fixing “agreement” is a legal question.

210. See part D.8. of this chapter for a discussion of structural form models vs. reduced form models.

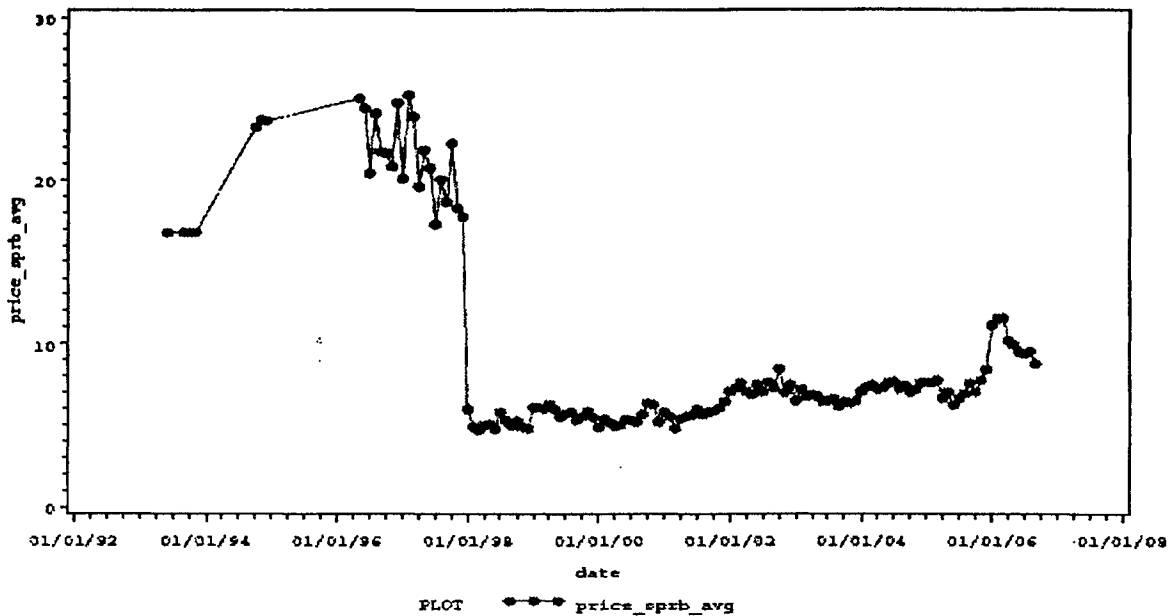
211. See WILLIAM H. GREENE, ECONOMETRIC ANALYSIS § 8.3 (2d ed. 1990).

212. The data source is FERC Form-423; Energy Velocity. Regulated utilities report the delivered price of their fuels in FERC Form-423. Energy Velocity estimates transportation costs and subtracts these from the

spot price at the time of delivery instead of at the time of actual sales. As a result, there is a lag between the reported spot price and the spot price at the time of actual sales that ranges from one quarter to one year. Therefore, although the alleged collusion occurred in the first quarter of 2001, the reported spot price did not increase until late 2001. This case study defines the before collusion period as before November 2001 and the during collusion period as between November 2001 and May 2005.

Before one can engage in econometric analysis of the data, it is important to review the data and check whether there are potential coding errors or other issues in the data that deserve further investigation. As discussed in section A, data do not have to be perfectly accurate to be useful in econometric analysis, but it is important to ensure that the data are reasonably reliable. To illustrate the importance of initial data review before running any regression analysis, the original data on the average SPRB coal price are plotted in Figure 4.

Figure 4: Average Price of SPRB Coal before Data Cleaning

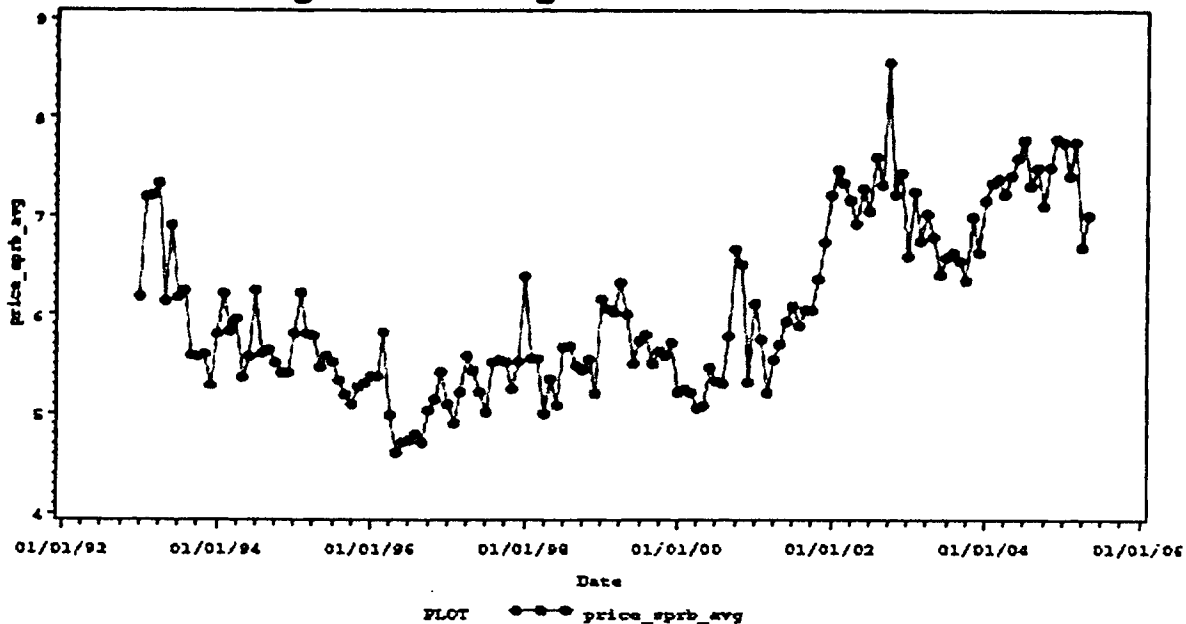


As can be seen clearly from Figure 4, something is not right with the SPRB coal price data before 1998. Further review of the data reveals

delivered costs to provide a comparable FOB mine price for each utility. The average of these monthly spot FOB prices provides data for our dependent variable.

that before 1998, the transportation cost from the coal mines to the utilities was in the column for the spot price and vice versa. Correcting these data and conducting some other cleaning of the data, such as estimating the spot prices that were missing, results in data on the corrected average spot price for SPRB coal that are presented in Figure 5.

Figure 5: Average Price of SPRB Coal



These prices represent the prices of SPRB coal delivered from January 1993 to May 2005. Through most of the 1990s, these prices display significant volatility, but they did not have any trend up or down. Beginning in November 2001, prices began increasing from average levels of around \$5.60/ton in the before period to over \$7.00/ton in the during period. The question is whether this increase in prices was caused by changes in demand, changes in costs, or changes in the behavior of mine operators.

To take into account the major demand and supply factors that potentially could affect the prices of the SPRB coal, data on these factors were obtained from public sources. Supply data include quality measures of SPRB coal (Btu content, ash percentage, and sulfur content); productivity of the mines that produce the SPRB coal as measured by the average tons of SPRB coal produced per person day; and costs of fuels that are used in the production of SPRB coal, including diesel fuel cost

and electricity cost.²¹³ Demand was measured using data on total electric power production.²¹⁴

3. Assumption of Liability and Causation

In performing a damage analysis, experts usually assume that the plaintiff has been found liable for the alleged anticompetitive conduct. Accordingly, this case study assumes that SPRB coal producers engaged in collusive behavior from November 2001 to May 2005. As discussed in section C, however, even assuming collusion did occur, one still needs to study whether the observed price increase was at least in part due to other demand or supply changes or could be attributed to the collusive behavior. To help answer this question and estimate the price increase that resulted from the assumed collusion, the dummy variable model and the prediction model under the before-during approach are used to isolate the price effects of the alleged collusive behavior.

4. Stationarity of the Time Series

As discussed in section D, when using time series data to conduct regression analysis, one should test whether the time series are stationary. If they are not stationary, different econometric techniques should be employed. To study whether nonstationarity is an issue in the time series data, Dickey-Fuller/Augmented Dickey-Fuller unit root tests were conducted²¹⁵ on the SPRB coal price data and certain independent variables.

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213. Time series data on capacity for SPRB coal production were not available. If the data were available, one may want to consider capacity in the analysis as well since it may affect the supply of SPRB coal.
214. The spot prices of SPRB coal were reported by the utilities at the time of delivery instead of at the time of sales, so there is a lag between the time when the prices were reported and the time when the prices were determined. This delay in reporting is not an issue for other independent variables, such as diesel cost, electricity cost, electricity production and SPRB coal productivity measurement. Therefore, in theory, one should lag these independent variables for certain periods and include these lagged variables as independent variables in the regression as well. However, this undertaking is beyond the scope of this illustration.
215. See MADDALA, *supra* note 14, at 548-52. In the Dickey-Fuller/Augmented Dickey-Fuller test, the null hypothesis is that the time series is nonstationary. *Id.* For a brief introduction to stationarity tests, see MADDALA, *supra* note 14, ch. 14.

The unit root test results indicate that the SPRB coal price data are stationary while some of the independent variables may be nonstationary, such as the diesel price series. However, as pointed out in section D, many unit root tests have a low ability to reject nonstationarity. Therefore, one should view the results of the tests within the context of the economic theory. In this case, if the dependent variable, SPRB coal price, is stationary, it is unlikely that the demand or supply factors that interact with each other to determine the SPRB coal price are nonstationary. If these demand or supply factors were nonstationary, they likely would drive the SPRB coal price to be nonstationary as well.

To further investigate the nonstationarity issue, a Dickey-Fuller/Augmented Dickey-Fuller test and a KPSS test were conducted on the residuals from the OLS regression where the SPRB coal price is the dependent variable and other demand and supply factors along with a collusion dummy variable are included as the explanatory variables.²¹⁶ These stationarity tests indicate that the residuals from the OLS regression are stationary. Since the SPRB coal price is stationary, this result again suggests that nonstationarity of other explanatory variables is not an issue in the case study. Thus, an OLS regression analysis was conducted to assess how the assumed collusion affected the SPRB coal price.

5. *The Before-During Approach with the Dummy Variable Model*

The dummy variable model under the before-during approach can be used to measure how assumed collusion changed the price of SPRB coal. The dummy variable is constructed by creating a variable equal to 0 before November 2001 (before period) and equal to 1 from November

216. The OLS regression model specification is discussed in more detail in part H.5. of this chapter. In conducting the Dickey-Fuller/Augmented Dickey-Fuller tests of the OLS regression residuals, one should use the Engle-Granger critical values instead of the standard Dickey-Fuller critical values because the Dickey-Fuller test of the residuals is a co-integration test. For more details on the Engle-Granger critical values, see WALTER ENDERS, *APPLIED ECONOMETRIC TIME SERIES* 335-42 (2d ed. 2003). However, the exact Engle-Granger critical values for these tests were not available because the number of independent variables in our regression exceeds those listed in the table of Engle-Granger critical values in Enders. Therefore, the KPSS test was also performed. In the KPSS test, the null hypothesis is that the time series is stationary. See MADDALA, *supra* note 14, ch. 14, for more details.

2001 onward (during period). Together with other explanatory variables, the model is specified as

$$\begin{aligned} \text{SPRB_price}_i = & \alpha + \beta_1 \cdot \text{Btu}_i + \beta_2 \cdot \text{Ash}_i + \beta_3 \cdot \text{lbsSO2}_i + \beta_4 \cdot \text{ton_prod}_i \\ & + \beta_5 \cdot \text{diesel_price}_i + \beta_6 \cdot \text{electricity_price}_i + \beta_7 \cdot \text{electricity_production}_i \\ & + \gamma \cdot \text{dummy}_i + \varepsilon_i \end{aligned}$$

where *Btu_i* is the Btu content of the SPRB coal, *Ash_i* is the ash percentage of the SPRB coal, *lbsSO2_i* is the pounds of sulfur per million Btu of energy in the coal, *ton_prod_i* is the average tons of SPRB coal produced per person day, *diesel_price_i* is the price of diesel, *electricity_price_i* is the price of electricity, *electricity_production_i* is the production of electricity, *dummy_i* is the dummy variable, and ε_i is the error term.

If the assumed collusion led to an increase in the price of SPRB coal, the coefficient on the dummy variable should be positive and statistically significant after controlling for the demand and supply factors that affect the price of SPRB coal. As discussed in part D of this chapter, valid hypothesis testing requires consistent estimates of the standard error of the coefficient estimates. Tests for serial correlation indicate that the error terms in the regression are serially correlated. To estimate the standard errors consistently, the Newey-West procedure was used to obtain the heteroskedasticity and autocorrelation consistent Newey-West standard errors.²¹⁷ The result of the OLS regression with the Newey-West standard errors is presented in Table 1.²¹⁸

217. See JEFFREY M. WOOLDRIDGE, *INTRODUCTORY ECONOMETRICS—A MODERN APPROACH* 410-13 (2d ed. 2003).

218. Obtaining the Newey-West standard error requires determining the maximum order of lag in advance. This illustration follows the practice of using the smallest integer greater than or equal to $T/4$ where T is the sample size. See GREENE, *supra* note 16, at 465.

Table 1: Regression Results for Dummy Variable Method

Dependent Variable:		Average Price of SPRB Coal		
R-squared:				0.7132
Adj R-squared:				0.6968
Number of Obs:				149
	Coefficient	Newey-West Standard Error	t-Statistic	P> t
Dummy variable for the alleged collusion period	1.2362	0.1798	6.8800	0.0000
Btu content of SPRB coal	-0.0013	0.0011	-1.1300	0.2590
Ash percentage of the SPRB coal	-1.0727	0.7633	-1.4100	0.1620
Pounds of sulfur per million Btu	-1.1665	1.5701	-0.7400	
Average tons of SPRB coal produced per person day	0.0015	0.0024	0.6200	0.5330
Price of diesel	0.0978	0.3283	0.3000	0.7660
Price of electricity	0.0497	0.0483	0.2400	0.8080
Production of electricity	4.33E-10	1.78E-09	0.2400	0.8080
Constant	20.7707	11.0578	1.8800	0.0620

As can be seen from Table 1, the variables that control for supply and demand changes are of the expected signs. Individually they are not statistically significant from zero at the 5 percent level, although they are statistically significant when taken together.²¹⁹ The coefficient on the dummy variable is positive and statistically significant. The regression result indicates that on average, SPRB coal prices were \$1.23/ton (or about 22%) higher in the during period compared to the before period, holding constant the exogenous variables.

As discussed in part E of this chapter, the dummy variable model assumes that the relationship between the dependent variable and the explanatory variables in the during period is the same as in the before period. To test this assumption, interaction terms involving the dummy variable and each of the other explanatory variables except the intercept term were added as explanatory variables in the model specification as

219. In a more detailed analysis, other measures of supply and demand might be considered or the estimating equation might be modified to better take into account more specific demand and supply considerations for each of the utilities.

well.²²⁰ The null hypothesis that all of the coefficients on the interaction terms are jointly equal to zero was tested. If this hypothesis is not rejected, then the data are consistent with the view that the relationship between the price of SPRB coal and the demand and supply factors did not change from the before period to the during period. If the null hypothesis is rejected, it suggests that the relationship has changed, either due to the alleged behavior or because of other reasons.²²¹

As shown in Table 2, the test rejected the null hypothesis that the relationship between the price of the SPRB coal and the demand and supply factors remained the same in the during period.²²² As discussed in part E of this chapter, even though the test suggests that the coefficients on the demand and supply factors in the during period have changed (contrary to the assumption of the dummy variable model), it does not necessarily mean that the coefficient estimate of the dummy variable will not provide any economically meaningful information. A possibility that should be investigated is that the restrictions on the coefficients of the demand and supply factors imposed by the dummy variable model may not have substantially affected the model's estimate of the effect of the alleged antitrust violation. Accordingly, the next section uses the prediction model to assess the impact of the assumed collusion on the price of SPRB coal.²²³

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220. These interaction terms are equal to zero in the pre-collusion period and equal to the respective values of the original explanatory variables during the collusion period. *See* MADDALA, *supra* note 14, at 307-10.
221. *See* part E.1. of this chapter.
222. To implement the joint F test correctly, again, the Newey-West covariance matrix is used. *See* WOOLDRIDGE, *supra* note 217.
223. When one fully interacts the dummy variable with other explanatory variables, one in fact is assuming that the regression equations in the before period and in the during period are completely different. *See* MADDALA, *supra* note 14, at 309. For example, the regression equation in the during period can have smaller intercept terms, hence a negative coefficient on the dummy variable, but higher slope terms. As a result, one cannot interpret the coefficient on the dummy variable as the measurement of the average impact of the alleged collusion anymore.

Table 2: Regression Results—Testing for Relationship Change in the Dummy Variable Method

Dependent Variable:		Average Price of SPRB Coal		
R-squared:				0.7417
Adj R-squared:				0.7126
Number of Obs:				149
	Coefficient	Newey-West Standard Error	t-Statistic	P> t
Dummy variable for the alleged collusion period	-57.0152	23.4560	-2.4300	0.0160
Btu content of SPRB coal	-0.0021	0.0016	-1.3000	0.1940
Ash percentage of the SPRB coal	-1.9244	0.8683	-2.2200	0.0280
Pounds of sulfur per million Btu	0.9237	1.7494	0.5300	0.5980
Average tons of SPRB coal produced per person day	0.0038	0.0025	1.5300	0.1280
Price of diesel	-0.0652	0.4501	-0.1400	0.8850
Price of electricity	0.0296	0.0860	0.3400	0.7310
Production of electricity	9.85E-10	2.32E-09	0.4300	0.6710
Interaction term for Btu content of SPRB coal	0.0056	0.0025	2.2600	0.0260
Interaction term for ash percentage of the SPRB coal	2.5203	1.2294	2.0500	0.0420
Interaction term for pounds of sulfur per million Btu	-5.2794	2.4693	-2.1400	0.0340
Interaction term for average tons of SPRB coal produced per person day	0.0038	0.0081	0.4700	0.6370
Interaction term for price of diesel	0.5961	0.5924	1.0100	0.3160
Interaction term for price of electricity	-0.0406	0.1036	-0.3900	0.6960
Interaction term for production of electricity	-6.07E-10	2.96E-09	-0.2100	0.8380
Constant	29.8307	14.1026	2.1200	0.0360
Joint F test for all interaction terms are equal to zero	F(7, 133) = 3.09		Prob>F = 0.0048	

6. The Before-During Approach with the Prediction Model

The prediction model assumes that the relationship between the price of SPRB coal and the demand and supply factors in the actual world before the alleged anticompetitive behavior would have remained the same but for the alleged behavior in the during period.²²⁴ If the relationship change found in the last section is all due to the collusive behavior, then the prediction model still would be valid.

To use the prediction model to estimate the impact of the assumed collusion on the price of SPRB coal, first run an OLS regression similar to the dummy variable model, but using the data in the before collusion period only and excluding the dummy variable from the explanatory variables. The result of this regression is presented in Table 3.

Table 3: Regression Results for Prediction Model

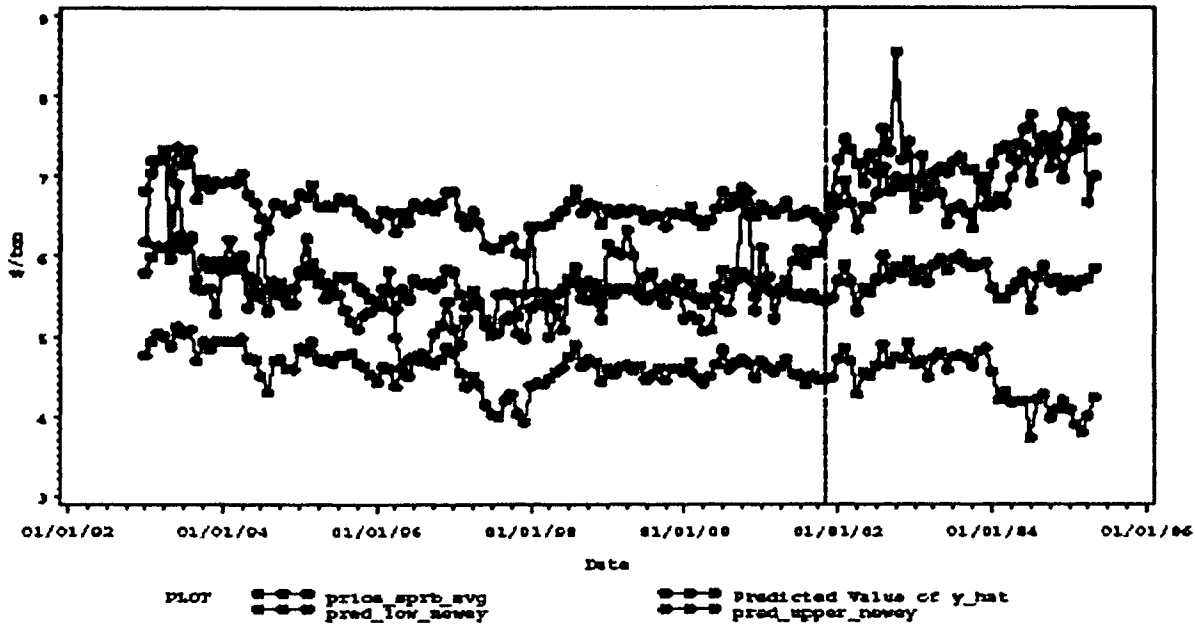
Dependent Variable:		Average Price of SPRB Coal		
R-squared:		0.2104		
Adj R-squared:		0.1540		
Number of Obs:		106		
	Coefficient	Newey-West Standard Error	t-Statistic	P> t
Btu content of SPRB coal	-0.0021	0.0015	-1.3300	0.1870
Ash percentage of the SPRB coal	-1.9244	0.8532	-2.2600	0.0260
Pounds of sulfur per million Btu	0.9237	1.7189	0.5400	0.5920
Average tons of SPRB coal produced per person day	0.0038	0.0025	1.5600	0.1220
Price of diesel	-0.0652	0.4423	-0.1500	0.8830
Price of electricity	0.0296	0.0845	0.3500	0.7270
Production of electricity	9.85E-10	2.28E-09	0.4300	0.6660
Constant	29.8307	13.8571	2.1500	0.0340

Given the coefficients from the before period OLS regression, the next step is to predict the price of SPRB coal in the but-for world without collusion, using the data on the explanatory variables in the during period. Figure 6 shows the actual price of the SPRB coal, the predicted price of the SPRB coal based on relationships estimated in the before

224. See the discussion in part A of this chapter.

period, and the lower and upper bound of the 95 percent confidence interval for the predicted price of the SPRB coal.²²⁵

Figure 6: Average vs. Predicted Price of SPRB Coal—Newey Standard Error



As can be seen in Figure 6, in every month in the during period, the actual price of the SPRB coal is higher than its predicted price. Moreover, in 28 of the 43 months in the during period, the actual price of the SPRB coal is higher than the upper bound of the 95 percent confidence interval for the predicted price of the SPRB coal. The average predicted price of the SPRB coal for the during period is \$5.72/ton, while the average actual price of the SPRB coal is \$7.16/ton in the during period. Therefore, according to the prediction model, the

225. The 95% confidence interval for the predicted value of the SPRB coal price means that there is a 95% "confidence" that the price of the SPRB coal in the but-for world will fall within this interval. See GREENE, *supra* note 16, § 6.6, for detailed discussion on how to construct a confidence interval for the predicted values. Constructing a confidence interval for the predicted value requires estimating the variances of the predicted value and the error term. The illustration uses the Newey-West standard errors in estimating the variance of the predicted value and uses the residuals from the OLS regression to estimate the variance of the error term.

assumed collusion resulted in a \$1.66/ton (or about a 30 percent) increase of the SPRB coal price. The prediction model results suggest a somewhat larger impact of the assumed collusion on the price of the SPRB coal compared to the \$1.23/ton (or 22 percent) increase of the SPRB coal price based on the dummy variable model. The results of both of these approaches are consistent with the alleged behavior causing higher prices. However, the prediction approach in this case would yield greater damages.

Given that the test shown in Table 2 above rejected the hypothesis that the relationship between the price of the SPRB coal and the demand and supply factors were the same in the before and during periods, the prediction approach would appear to be the more reliable quantification of the impact of the hypothesized conspiracy. However, to determine if the prediction approach is clearly superior, one could test the prediction approach for a portion of the period before the alleged anticompetitive behavior, as discussed above in part D. If it predicted prices before the alleged anticompetitive act reasonably accurately, then the prediction approach presumably would be more accurate in this example. If it does not predict prices before the alleged anticompetitive act that well, then it would be more accurate to say that the impact of the alleged anticompetitive behavior likely falls in the range indicated by the two methodologies.

7. *Benchmark Approach*

In a benchmark analysis, the first question is whether there are any products that might provide a reasonably good benchmark. As discussed in part F above, the benchmark should be (1) subject to similar supply and demand factors as SPRB coal and (2) unaffected by alleged anticompetitive acts. This illustration will use the price of bituminous coal (BIT).²²⁶

An econometric analysis using the benchmark approach can be implemented in several ways. To keep the illustration relatively simple,

226. Utilities also purchase this coal to generate electricity, and so BIT's prices likely reflect many of the same cost and demand factors. There is no allegation of the conspiracy extending to BIT. Moreover, most utilities cannot substitute SPRB and BIT, at least in the short run, so the alleged conspiracy should have no direct effect on the price of BIT. In *Federal Trade Commission v. Arch Coal, Inc.*, the court found "the relevant product market is no broader and no narrower than SPRB coal." *FTC v. Arch Coal, Inc.*, 329 F. Supp. 2d 109, 123 (2004).

the impact of the alleged conspiracy is tested using the dummy variable model discussed above. Most of the supply and demand factors used in the before-during approaches are replaced by the benchmark BIT price, since it should be capturing those effects.²²⁷ The model includes the same dummy variable, as well as three variables to control for the quality of SPRB coal. The estimating equation is:

$$\begin{aligned} \text{SPRB_price}_t = & \alpha + \beta_1 \cdot \text{Btu}_t + \beta_2 \cdot \text{Ash}_t + \beta_3 \cdot \text{lbsSO}_2_t \\ & + \beta_4 \cdot \text{BIT_price}_t + \gamma \cdot \text{dummy}_t + \varepsilon_t \end{aligned}$$

where BIT_price_t is the price of BIT coal in period t . The results of this regression are shown in Table 4.

Table 4: Regression Results for Benchmark Approach

Dependent Variable:	Average Price of SPRB Coal			
R-squared:	0.7191			
Adj R-squared:	0.7093			
Number of Obs:	149			
	Coefficient	Newey-West Standard Error	t-Statistic	P> t
Dummy variable for the alleged collusion period	1.1527	0.1867	6.1700	0.0000
Btu content of SPRB coal	-0.0005	0.0010	-0.4800	0.6300
Ash percentage of the SPRB coal	-1.1329	0.6171	-1.8400	0.0680
Pounds of sulfur per million Btu	-1.1130	1.3201	-0.8400	0.4010
Average tons of SPRB coal produced per person day	0.0063	0.0028	2.2800	0.0240
Constant	15.7235	9.7266	1.6200	0.1080

As can be seen in Table 4, the BIT price is of the expected sign (positive) and is statistically significant. The results show a statistically significant impact on SPRB prices in the period during the alleged conspiracy, again consistent with the alleged conspiracy increasing price. This specification of the benchmark dummy variable model estimates a \$1.15/ton (or about 21 percent) increase in the SPRB coal price, which is somewhat smaller than the reduced form estimates in the before-during

227. The supply and demand factors could still be included in the model, and if they affect the benchmark differently than the price in question, including them will generally improve the model.

models using either the dummy variable or prediction approaches. Although BIT should not have been directly affected by the alleged anticompetitive act related to SPRB coal, it is possible that the customers who bought BIT faced substantially different supply and demand conditions than the plants purchasing the SPRB coal. In deciding whether the benchmark approach yields a more accurate quantification of the alleged anticompetitive behavior than the dummy variable or prediction approaches, one would need to analyze in more depth the comparability of the exogenous influences on BIT and SPRB to determine how good a benchmark BIT is for SPRB. If there are substantial questions about how comparable BIT is to SPRB, then the other approaches likely would yield more accurate estimates.