EXHIBIT Q

OMNIBUS BROWN DECLARATION

SETS OF POSTERIOR MEANS WITH BOUNDED VARIANCE PRIORS

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The matrix weighted average $(H + V^{-1})^{-1}Hb$, where H and V are symmetric positive definite matrices and b is a vector, is shown to lie in one ellipsoid if V is bounded from below, $V_* \le V$, another ellipsoid if V is bounded from above, $V \le V^*$, and another ellipsoid if V is bounded from above and below, $V_* \le V \le V^*$. These results are applied to bound the posterior mean vector of the normal linear regression model.

1. INTRODUCTION

A BOUND FOR THE POSTERIOR MEAN VECTOR in a normal linear regression model with the prior location given but the prior variance matrix free has been provided by Chamberlain and Leamer [1]. In this paper, I generalize this bound by assuming that the prior variance matrix is constrained to lie between a minimum variance matrix and a maximum variance matrix.

The construction of a multivariate prior distribution taxes the ability and patience of many who might otherwise use the Bayesian tools. The choice of an exact prior covariance matrix V is ordinarily very costly, and the methods by which V is elicited make many people uncomfortable.[2] The results in this paper are intended to reduce the cost and to increase the comfort in applying Bayesian methods.

2. RESULTS

A posterior mean for the k-variable linear regression model takes the form

$$(1) \qquad \hat{\beta}(V) \equiv E(\beta \mid Y, X, \sigma^2, V) = (H + V^{-1})^{-1}Hb$$

where

$$H = \sigma^{-2}X'X,$$
$$Hb = \sigma^{-2}X'Y.$$

X is a $(T \times k)$ matrix, Y is a $(T \times 1)$ vector, V is a $(k \times k)$ symmetric positive definite matrix, and σ^2 is a positive scalar. This equation results from the assumption that Y is normally distributed with mean vector $X\beta$ and covariance matrix $\sigma^2 I_T$, and β is normally distributed with mean vector zero and covariance matrix V.

As a practical matter, there are some settings in which the prior can be taken to be normal with a known location, but there are very few (if any) settings in which the covariance matrix V can sensibly be taken as given. A statistician may

[2] As an example see Kadane et al. [2].

ance σ^2 is concentrated on its estimator s^2, formula (6) defines the posterior mean with $H = s^{-2}X'X$. This formula implies the following estimated equation:

$$M = -.45 - .58PC - .10PX - .72T - .03W$$
$$+ .57POOR + .14NW - .02URB + .18YOUTH.$$

This set of estimated coefficients falls on the 89 per cent classical confidence ellipsoid and therefore is within the traditionally accepted 95 per cent region. The estimated effect of executions is $-.10$ with a standard error of .10; or, each execution deters 7.4 murders with a standard error of 7.7.

Various bounds are reported on Table II. The extreme bounds are found in the upper right-hand corner of this matrix. The estimate of the execution effect can range from -54.9 to 44.2 depending on the prior which is used. The diagonal elements of this matrix contain estimates of the execution effect as the prior covariance matrix is scaled up and down. The upper left-hand corner contains the prior estimate, zero. The lower right-hand corner contains the least-squares estimate, -10.7, the negative sign indicating that executions deter murders. The center of the matrix (in a box) contains the estimate, -7.36, based on the prior covariance matrix described in Table I. Moving from this box toward the upper-right we find estimates with the lower variance matrix less than, and the upper variance matrix greater than, the representative variance matrix. The figures which are in a box are based on priors with covariance matrices between $(\frac{1}{2})^2$ and 2^2 times the input matrix. Because this interval from -28.0 to 14.1 contains the origin, and because I am unable to define more precisely my prior, I am forced to conclude that these data are not useful for estimating the effects of executions. Of course, other analysts with more sharply defined priors possibly based on other data sets could find these data useful. In particular, as can be seen in Table II, if you are sure that your prior is more diffuse than mine, in the sense that your covariance matrix is certainly greater than 2^2 times the matrix defined in Table I, then you can only obtain a negative estimate, and can conclude that executions deter murders.

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REFERENCES

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[3] LEAMER, E. E.: *Specification Searches*. New York: Wiley, 1978.
[4] McMANUS, W. S.: "Murder and Capital Punishment: A Bayesian Analysis," Department of Economics, UCLA, mimeographed, 1980.