Exhibit A

Dr. Chris Mack Curriculum Vitae

Chris A. Mack

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EDUCATION

University of Texas, Austin, TX
Doctorate in Chemical Engineering — December 1998
Thesis Title: "Modeling Solvent Effects in Optical Lithography"

University of Maryland, College Park, MD
Master of Science in Electrical Engineering — December 1989

Rose-Hulman Institute of Technology, Terre Haute, IN
Bachelor of Science degrees in Physics, Electrical Engineering, Chemistry, and Chemical Engineering — May 1982

EXPERIENCE

Lithoguru.com, Austin, TX

11/05 – present *Gentleman Scientist*

- Pursuing intellectual interests, research, writing, and teaching, as reflected on the website www.lithoguru.com.
- Current major research interest: developing an approximate analytical stochastic model of lithography line-edge roughness.
- Consulting in the fields of optics and semiconductor lithography, including legal expert witness services and business consultation.

University of Texas at Austin, Austin, TX

8/91 – present *Adjunct Faculty (part time)*

- Teaching graduate and undergraduate courses in the Electrical Engineering and Chemical Engineering departments. Graduate courses include Semiconductor Microlithography, Chemical Processes for Microelectronics, and Fourier Optics. Undergraduate courses include Electronic Circuits, Solid State Electronics, and Modern Optics. Served on the committees of numerous PhD dissertations.
- Teaching loads have varied but have averaged 1 – 2 courses per semester.

University of Notre Dame, South Bend, IN

8/06 – 12/06 *Melchor Visiting Chair Professor*

- Teaching two graduate courses in the Electrical Engineering department: Semiconductor Microlithography, and Data Analysis and Modeling in the Real World.

KLA-Tencor, Austin, TX

2/00 – 11/05 *Vice President of Lithography Technology*

- Provided strategic vision in all lithography related products for KLA-Tencor, a $2B Fortune 500 supplier of equipment to the semiconductor industry.
- Directed research efforts for four product divisions across two continents, including lithography simulation, optical and SEM critical dimension metrology, and optical overlay metrology. Obtained funding and managed resource allocation and strategic planning for critical long-term projects.
- Provided and oversaw successful turn-around strategies for two failing product lines.
- Provided internal consulting services in lithography to other KLA-Tencor divisions.
- Oversaw the acquisition of FINLE Technologies by KLA-Tencor and its transition to a successful product division.

FINLE Technologies, Austin, TX

2/90 – 2/00 *CEO, President and Chief Technical Officer*

- Founded company in 1990, pursuing it full time by the end of 1991.
- Responsible for overall corporate management, vision, strategic planning, technical direction, budgeting, new product development, and lithography research. Grew the company from one person and $60,000 in revenue in 1990 to 25 people and $2.5M in revenue in 2000.
- Developed the industry standard PROLITH Toolkit of lithography simulation software and the ProDATA suite of data analysis software.
- Provided consulting services to the semiconductor industry.
- Taught numerous short courses on optical lithography.

SEMATECH, Austin, TX

8/90 – 12/91 *Lithography Engineer*

- As an assignee of the department of defense to SEMATECH, provided lithography expertise to SEMATECH on a variety of different projects, including modeling and process development for deep-UV resist systems, processes optimization of the i-line production process, advanced development activities in phase-shifting mask technologies, and lithographic lens design.
- Taught short-term and long-term courses on lithography to SEMATECH staff and assignees.

National Security Agency, Fort Meade, MD

11/82 – 8/90 *Senior Engineer - Lithography*

- As a member of the Microelectronics Research Laboratory (MRL), was tasked with performing research for present and future agency needs in the area of microlithography for semiconductor processing. This work provided a unique blend of theoretical research (e.g., a mechanism for the development reaction, diffraction theory for proximity printing and aerial imaging) and experimental work (measurement of resist properties, model verification). Performed numerous practical and theoretical studies, e.g., resist coating uniformity on wafer tracks, mask bias effects for step-and-repeat printing, exposure optimization, image reversal techniques, and focus effects for submicron lithography. The results of this work have been published in numerous journals and presented at technical conferences, including invited papers at international conferences in Japan and Europe.

COURSES TAUGHT AT THE UNIVERSITY OF TEXAS AT AUSTIN

EE 411	**Circuit Theory (undergraduate)**
EE 323	**Network Theory II (undergraduate)**
EE 325	**Electromagnetic Engineering (undergraduate)**
EE 338	**Electronic Circuits I (undergraduate)**
EE 339	**Solid State Electronics (undergraduate)**
CHE 323	**Chemical Engineering for Microelectronics (undergraduate)**
PHY 333/EE 347	**Modern Optics (undergraduate)**
SSC306	**Statistics in Market Analysis (undergraduate)**
EE 383P	**Fourier Optics (graduate)**
EE 396K/CHE 385C	**Semiconductor Microlithography (graduate)**
CHE 395C	**Chemical Processes for Microelectronics (graduate)**
SSC380D	**Statistical Methods II (graduate)**

COURSES TAUGHT AT THE UNIVERSITY OF NOTRE DAME

EE 60598	**Semiconductor Microlithography (graduate)**
EE 60596	**Data Analysis and Modeling in the Real World (graduate)**

AWARDS

SPIE Frits Zernike Award for Microlithography, for contributions in lithography modeling and education, 2009

SEMI Award for North America, for contributions in lithography modeling and education, 2003

Best Paper Award, *18th Annual BACUS Symposium on Photomask Technology and Management*, 1998.

INDUSTRIAL AND PROFESSIONAL SOCIETIES

Member of the Board of Trustees, Rose-Hulman Institute of Technology, 2008 – present

Member of the Board of Advisors to the Physics Department, Rose-Hulman Institute of Technology, 2000 – 2008

Member of the Board of Advisors to the Chemistry Department, Rose-Hulman Institute of Technology, 2003 – 2008

Member of the Board of Advisors to the MEMS Laboratory, Rose-Hulman Institute of Technology, 2004 – 2008

Fellow of SPIE

Fellow of IEEE

Member of the Optical Society of America

Chairman of the Lithography Technical Working Group of the Optical Society of America, 1992 – 1996.

Conference Chair, *Microlithographic Techniques in IC Fabrication*, SPIE Conference, 1997 and 2000, Singapore

Conference Chair, *Lithography for Semiconductor Manufacturing*, SPIE Conference, 1999 and 2001, Edinburgh, Scotland

Conference Chair, *Advanced Microlithography Technology*, SPIE Conference, 2007, Beijing, China

Plenary Speaker, *SPIE 2003 Symposium on Microlithography*.

Member of the Board of Advisors to *Semiconductor International* magazine, 1993 – 2004

Associate Editor, *Journal of Microlithography, Microfabrication, and Microsystems (JM3)*, 2002 – present

Member of the Board of Advisors to *Microlithography World* magazine, 2003 – 2008

Contributing Columnist for *Microlithography World* magazine, 1993 – 2008

OTHER PROFESSIONAL EXPERIENCE

Expert witness consulting in the field of lithography and semiconductor design and manufacturing

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PUBLICATIONS

Books

Chris A. Mack, Fundamental Principles of Optical Lithography: The Science of Microfabrication, John Wiley & Sons (in press, to be published November, 2007)

Chris A. Mack, Field Guide to Optical Lithography, SPIE Field Guide Series Vol. FG06, (Bellingham, WA: 2006). Also available in Japanese.

C. A. Mack, Inside PROLITH: A Comprehensive Guide to Optical Lithography Simulation, FINLE Technologies (Austin, TX: 1997). – Out of Print.

Book Chapters

C.A. Mack, “Microlithography”, Chapter 9, Semiconductor Manufacturing Handbook, Hwaiyu Geng, Ed., McGraw Hill (New York: 2005).

Contributed “Microlithography” entry for the McGraw Hill Encyclopedia of Science & Technology, 9th Edition (2005).

Contributed lithography terms for: Comprehensive Dictionary of Electrical Engineering, Phillip A. Laplante, Ed., (CRC Press and IEEE Press, 1999).

C.A. Mack, “Optical Lithography Modeling,” Chapter 2, Microlithography Science and Technology, J. R. Sheats and B. W. Smith, editors, Marcel Dekker (New York: 1998) pp. 109-170.

C.A. Mack and A. R. Neureuther, “Optical Lithography Modeling,” Chapter 7, Handbook of Microlithography, Micromachining, and Microfabrication, Volume 1: Microlithography, P. Rai-Choudhury, editor, SPIE Press (Bellingham, WA: 1997) pp. 597-680.

R. Hershel and C. A. Mack, “Lumped Parameter Model for Optical Lithography,” Chapter 2, Lithography for VLSI, VLSI Electronics - Microstructure Science Volume 16, R. K. Watts and N. G. Einspruch, eds., Academic Press (New York:1987) pp. 19-55.

Conference Chair/Proceedings Editor

Quantum Optics, Optical Data Storage, and Advanced Microlithography, Proceedings of SPIE Volume 6827 (2007)
Editors: Guangcan Guo; Songhao Liu; Guofan Jin; Kees A. Schouhamer Immink; Keiji Shono; Chris A. Mack; Jinfeng Kang; Jun-en Yao

Lithography for Semiconductor Manufacturing II, Proceedings of SPIE Volume 4404 (2001)
Editors: Chris A. Mack; Tom Stevenson

Microlithographic Techniques in Integrated Circuit Fabrication II, Proceedings of SPIE Volume 4226 (2000)
Editors: Chris A. Mack; XiaoCong Yuan

Lithography for Semiconductor Manufacturing, Proceedings of SPIE Volume 3741 (1999)
Editors: Chris A. Mack; Tom Stevenson

Microlithographic Techniques in IC Fabrication, Proceedings of SPIE Volume 3183 (1997)
Editors: Soon Fatt Yoon; Raymond Yu; Chris A. Mack

Patents

U.S. Patent 5,363,171, Photolithography exposure tool and method for in situ photoresist measurements and exposure control, November 8, 1994

U.S. Patent 6,968,253, Computer-implemented method and carrier medium configured to generate a set of process parameters for a lithography process, November 22, 2005

U.S. Patent 7,075,639, Method and Mark for Metrology of Phase Errors on Phase Shift Masks, July 11, 2006

U.S. Patent 7,142,941, Computer-implemented Method and Carrier Medium Configures to Generate a Set of Process Parameters and/or a List of Potential Causes of Deviations for a Lithography Process, November 28, 2006.

U.S. Patent 7,297,453, Systems and Methods for Mitigating Variances on a Patterned Wafer Using a Prediction Model, November 20, 2007.

U.S. Patent 7,300,725, Method for Determining and Correcting Reticle Variations, November 27, 2007.

U.S. Patent 7,300,729, Method for Monitoring a Reticle, November 27, 2007.

U.S. Patent 7,303,842, Systems and Methods for Modifying a Reticle's Optical Properties, December 4, 2007.

U.S. Patent 7,352,453, Method for Process Optimization and Control by Comparison Between 2 or More Measured Scatterometry Signals, April 1, 2008.

U.S. Patent 7,368,208, Measuring Phase Errors on Phase Shift Masks, May 6, 2008.

U.S. Patent 7,382,447, Method for Determining Lithographic Focus and Exposure, June 3, 2008.

U.S. Patent 7,528,953, Target Acquisition and Overlay Metrology Based on Two Diffracted Orders Imaging, May 5, 2009.

U.S. Patent 7,566,517, Feature Printability Optimization by Optical Tool, July 28, 2009.

U.S. Patent 7,804,998, Overlay Metrology and Control Method, September 28, 2010.

Refereed Papers

1. C. A. Mack, "Analytical Expression for the Standing Wave Intensity in Photoresist", *Applied Optics*, Vol. 25, No. 12 (15 June 1986) pp. 1958-1961.
2. C. A. Mack, "Development of Positive Photoresists," *Journal of the Electrochemical Society*, Vol. 134, No. 1 (Jan. 1987) pp. 148-152.
3. C. A. Mack, "Contrast Enhancement Techniques for Submicron Optical Lithography," *Journal of Vacuum Science & Technology*, Vol. A5, No. 4 (Jul./Aug. 1987) pp. 1428-1431.
4. C. A. Mack, "Dispelling the Myths about Dyed Photoresist," *Solid State Technology*, Vol. 31, No. 1 (Jan. 1988) pp. 125-130.
5. C. A. Mack, "Absorption and Exposure in Positive Photoresist," *Applied Optics*, Vol. 27, No. 23 (1 Dec. 1988) pp. 4913-4919.
6. C. A. Mack and P. M. Kaufman, "Mask Bias in Submicron Optical Lithography," *Journal of Vacuum Science & Technology*, Vol. B6, No. 6 (Nov./Dec. 1988) pp. 2213-2220.
7. C. A. Mack, "Understanding Focus Effects in Submicron Optical Lithography", *Optical Engineering*, Vol. 27, No. 12 (1 Dec 1988) pp. 1093-1100.
8. C. A. Mack, "Lithographic Optimization Using Photoresist Contrast," *Microelectronics Manufacturing Technology*, Vol. 14, No. 1 (Jan. 1991) pp. 36-42.
9. D. H. Ziger and C. A. Mack, "Generalized Approach toward Modeling Resist Performance," *AICHE Journal*, Vol. 37, No. 12 (Dec 1991) pp. 1863-1874.
10. C. A. Mack, E. Capsuto, S. Sethi, and J. Witowski, "Modeling and Characterization of a 0.5µm Deep Ultraviolet Process," *Journal of Vacuum Science & Technology*, Vol. B 9, No. 6 (Nov / Dec 1991) pp. 3143-3149.

11. D. Ziger, C. A. Mack, and R. Distasio, “Generalized Characteristic Model for Lithography: Application to Negative Chemically Amplified Resists,” *Optical Engineering*, Vol. 31, No. 1 (1 Jan 1992) pp.98-104.
12. C. A. Mack and J. E. Connors, “Fundamental Differences Between Positive and Negative Tone Imaging,” *Microlithography World*, Vol. 1, No. 3 (Jul/Aug 1992) pp. 17-22.
13. D. W. Johnson and C. A. Mack, Modeling the Continuing Realm of Optical Lithography” *Semiconductor International*, Vol. 15, No. 6 (June 1992) pp. 134-139.
14. C. A. Mack, “New Kinetic Model for Resist Dissolution,” *Journal of the Electrochemical Society*, Vol. 139, No. 4 (Apr. 1992) pp. L35-L37.
15. C. A. Mack, “Understanding Focus Effects in Submicrometer Optical Lithography: a Review,” *Optical Engineering*, Vol. 32, No. 10 (Oct. 1993) pp. 2350-2362.
16. E. W. Charrier, C. J. Progler and C. A. Mack, “Comparison of Simulated and Experimental CD-Limited Yield for a Submicron I-Line Process,” *Solid State Technology*, Vol. 38, No. 11 (Nov. 1995) pp. 105-112.
17. C. A. Mack, “Lithographic Effects of Acid Diffusion in Chemically Amplified Resists,” Microelectronics Technology: Polymers for Advanced Imaging and Packaging, ACS Symposium Series 614, E. Reichmanis, C. Ober, S. MacDonald, T. Iwayanagi, and T. Nishikubo, eds., ACS Press (Washington: 1995) pp. 56-68.
18. C. A. Mack, “Evaluating Proximity Effects Using 3-D Optical Lithography Simulation,” *Semiconductor International* (July, 1996) pp. 237-242.
19. C. A. Mack, “Trends in Optical Lithography,” *Optics and Photonics News* (April, 1996) pp. 29-33.
20. C. A. Mack, G. E. Flores, W. W. Flack, and E. Tai, “Lithographic Modeling Speeds Thin-Film-Head Development,” *Data Storage* (May/June, 1996) pp. 55-58.
21. C. A. Mack, “Reducing Proximity Effects in Optical Lithography,” *Japanese Journal of Applied Physics*, Vol. 35 (1996) pp. 6379-6385.
22. C. A. Mack and G. Arthur, “Notch Model for Photoresist Dissolution,” *Electrochemical and Solid State Letters*, Vol. 1, No. 2, (August, 1998) pp. 86-87.
23. C. A. Mack, K. E. Mueller, A. B. Gardiner, J. P. Sagan, R. R. Dammel, and C. G. Willson “Modeling Solvent Diffusion in Photoresist,” *Journal of Vacuum Science & Technology*, Vol. B16, No. 6, (Nov., 1998) pp. 3779-3783.
24. C. A. Mack, D. A. Legband, S. Jug, “Data Analysis for Photolithography” *MicroElectronic Engineering*, Vol. 46, Issues 1-4 (May 1999) pp. 65-68.
25. C. A. Mack, “Electron Beam Lithography Simulation for Mask Making” *MicroElectronic Engineering*, Vol. 46, Issues 1-4 (May 1999) pp. 283-286.
26. Sergey Babin, Igor Yu. Kuzmin and Chris A. Mack, “Comprehensive Simulation of Electron-beam Lithography Processes Using PROLITH 3/D and TEMPTATION Software Tools,” *MicroElectronic Engineering,* Volumes 57-58 (September 2001) pp. 343-348.
27. J. Byers, C. Mack, R. Huang, S. Jug, “Automatic Calibration of Lithography Simulation Parameters Using Multiple Data Sets,” *MicroElectronic Engineering,* Volumes 61-62 (July 2002) pp. 89-95.

28. Chris A. Mack, “Charting the Future (and Remembering the Past) of Optical Lithography Simulation,” *Journal of Vacuum Science & Technology*, Vol. B 23, No. 6 (Nov / Dec 2005) pp. 2601-2606.
29. C. A. Mack, "Accuracy, speed, new physical phenomena: The future of litho simulation," *Solid State Technology*, February, 2006.
30. C. A. Mack, “The Future of Semiconductor Lithography: After Optical, What Next?”, *Future Fab International*, Vol. 23 (7/9/2007).
31. Chris A. Mack, “Fab Future”, *SPIE Professional* (Oct. 2008) pp. 10-11.
32. Chris A. Mack, “Seeing Double”, *IEEE Spectrum* (Nov. 2008) pp. 46-51.
33. C. Mack, “Stochastic approach to modeling photoresist development”, *Journal of Vacuum Science & Technology*, Vol. B27, No. 3 (May/Jun. 2009) pp. 1122-1128.
34. C. A. Mack, “Stochastic Modeling in Lithography: Autocorrelation Behavior of Catalytic Reaction-Diffusion Systems,” *Journal of Micro/Nanolithography, MEMS, and MOEMS*, Vol. 8, No. 2 (Apr/May/Jun 2009) p. 029701.
35. C. A. Mack, “Stochastic Modeling in Lithography: The Use of Dynamical Scaling in Photoresist Development,” *Journal of Micro/Nanolithography, MEMS, and MOEMS*, Vol. 8, No. 3 (Jul/Aug/Sep 2009) p. 033001.
36. Chris Mack, “A Simple Model of Line-Edge Roughness”, *Future Fab International*, Vol. 34 (July 14, 2010).
37. C. A. Mack, “Stochastic modeling of photoresist development in two and three dimensions”, *Journal of Micro/Nanolithography, MEMS, and MOEMS* , Vol. 9, No. 4 (Oct-Dec, 2010) p. 041202.

Invited Papers

1. C. A. Mack, “Lithographic Simulation: A Review,” *Lithographic and Micromachining Techniques for Optical Component Fabrication, Proc.,* SPIE Vol. 4440 (2001) pp. 59-72.
2. Chris A. Mack, “The End of the Semiconductor Industry as We Know It,” *Optical Microlithography XVI*, Plenary Address, SPIE Vol. 5040 (2003) pp. xxi-xxxi.
3. C. A. Mack, “The New, New Limits of Optical Lithography,” *Emerging Lithographic Technologies VIII, Proc*., SPIE Vol. 5374 (2004) pp. 1-8.
4. Chris A. Mack, “Thirty Years of Lithography Simulation,” *Optical Microlithography XVIII, Proc*., SPIE Vol. 5754-1 (2005), pp. 1-12.
5. C. A. Mack, “What's So Hard About Lithography?,” *presented at the ICMTS* (March, 2006), available at http://www.lithoguru.com/scientist/papers_recent.html.

Contributed Papers

1. C. A. Mack, “PROLITH: A Comprehensive Optical Lithography Model,” *Optical Microlithography IV, Proc.*, SPIE Vol. 538 (1985) pp. 207-220.
2. C. A. Mack and R. T. Carback, “Modeling the Effects of Prebake on Positive Resist Processing,” *Kodak Microelectronics Seminar, Interface ’85, Proc.*, (1985) pp. 155-158.
3. C. A. Mack, “Advanced Topics in Lithography Modeling,” *Advances in Resist Technology and Processing III, Proc.*, SPIE Vol. 631 (1986) pp. 276-285.

4. C. A. Mack, A. Stephanakis, R. Hershel, “Lumped Parameter Model of the Photolithographic Process,” *Kodak Microelectronics Seminar, Interface ’86, Proc.*, (1986) pp. 228-238.
5. C. A. Mack, “Photoresist Process Optimization,” *KTI Microelectronics Seminar, Interface ’87, Proc.*, (1987) pp. 153-167.
6. T. Brown and C. A. Mack, “Comparison of Modeling and Experimental Results in Contrast Enhancement Lithography,” *Advances in Resist Technology and Processing V, Proc.*, SPIE Vol. 920 (1988) pp. 390-403.
7. C. A. Mack, “Understanding Focus Effects in Submicron Optical Lithography,” *Optical/Laser Microlithography, Proc.*, SPIE Vol. 922 (1988) pp. 135-148.
8. D. H. Ziger and C. A. Mack, “Lithographic Characterization of a Rapid Ammonia Catalyzed Image Reversal Process,” *KTI Microelectronics Seminar, Interface ’88, Proc.*, (1988) pp. 165-175.
9. C. A. Mack and P. M. Kaufman, “Understanding Focus Effects in Submicron Optical Lithography, part 2: Photoresist effects,” *Optical/Laser Microlithography II, Proc.*, SPIE Vol. 1088 (1989) pp. 304-323.
10. C. A. Mack and P. M. Kaufman, “Focus Effects in Submicron Optical Lithography, Optical and Photoresist Effects,” *The International Congress on Optical Science & Engineering*, Proc., Paris, France, SPIE Vol. 1138 (1989) pp. 88-105.
11. C. A. Mack, “Optimum Stepper Performance Through Image Manipulation,” *KTI Microelectronics Seminar, Interface ’89, Proc.*, (1989) pp. 209-215.
12. C. A. Mack, “Algorithm for Optimizing Stepper Performance Through Image Manipulation,” *Optical/Laser Microlithography III, Proc.*, SPIE Vol. 1264 (1990) pp. 71-82.
13. C. A. Mack, “Lithographic Optimization Using Photoresist Contrast,” *KTI Microlithography Seminar, Interface ’90, Proc.*, (1990) pp. 1-12.
14. P. Trefonas and C. A. Mack, “Exposure Dose Optimization for a Positive Resist Containing Poly-functional Photoactive Compound,” *Advances in Resist Technology and Processing VIII, Proc.*, SPIE Vol. 1466 (1991) pp. 117-131.
15. D. Ziger, C. A. Mack, and R. Distasio, “The Generalized Characteristic Model for Lithography: Application to Negative Chemically Amplified Resists,” *Advances in Resist Technology and Processing VIII, Proc.*, SPIE Vol. 1466 (1991) pp. 270-282.
16. C. A. Mack, “Fundamental Issues in Phase-Shifting Mask Technology,” *KTI Microlithography Seminar, Interface ’91, Proc.*, (1991) pp. 23-35.
17. M. A. Toukhy, S. G. Hansen, R. J. Hurditch, and C. A. Mack, “Experimental Investigation of a Novel Dissolution Model,” *Advances in Resist Technology and Processing IX, Proc.*, SPIE Vol. 1672 (1992) pp. 286-296.
18. C. A. Mack, “Understanding Focus Effects in Submicron Optical Lithography, part 3: Methods for Depth-of-Focus Improvement,” *Optical/Laser Microlithography V, Proc.*, SPIE Vol. 1674 (1992) pp. 272-284.
19. C. A. Mack and J. E. Connors, “Fundamental Differences Between Positive and Negative Tone Imaging,” *Optical/Laser Microlithography V, Proc.*, SPIE Vol. 1674 (1992) pp. 328-338.

20. D. W. Johnson and C. A. Mack, “I-line, DUV, VUV, or X-Ray?” *Optical/Laser Microlithography V, Proc.*, SPIE Vol. 1674 (1992) pp. 486-498.
21. C. A. Mack, “Simple Method for Rim Shifter Design: The Biased Self-Aligned Rim Shifter,” *12th Annual BACUS Symposium, Proc.*, SPIE Vol. 1809 (1992) pp. 229-236.
22. N. Thane, C. A. Mack, and S. Sethi, “Lithographic Effects of Metal Reflectivity Variations,” *Integrated Circuit Metrology, Inspection, and Process Control VII, Proc.*, SPIE Vol. 1926 (1993) pp. 483-494.
23. C. A. Mack, “Phase Contrast Lithography,” *Optical/Laser Microlithography VI, Proc.*, SPIE Vol. 1927 (1993) pp. 512-520.
24. C. A. Mack, “Optimization of the Spatial Properties of Illumination,” *Optical/Laser Microlithography VI, Proc.*, SPIE Vol. 1927 (1993) pp. 125-136.
25. P. M. Mahoney and C. A. Mack, “Cost Analysis of Lithographic Characterization: An Overview,” *Optical/Laser Microlithography VI, Proc.*, SPIE Vol.1927 (1993) pp. 827-832.
26. C. A. Mack, “Designing the Ultimate Photoresist,” *OCG Microlithography Seminar, Interface ’93, Proc.*, (1993) pp. 175-191.
27. G. E. Flores, W. W. Flack, E. Tai, and C. A. Mack, “Lithographic Performance in Thick Photoresist Applications,” *OCG Microlithography Seminar, Interface ’93, Proc.*, (1993) pp. 41-60.
28. C. A. Mack, D. P. DeWitt, B. K. Tsai, and G. Yetter, “Modeling of Solvent Evaporation Effects for Hot Plate Baking of Photoresist,” *Advances in Resist Technology and Processing XI, Proc.*, SPIE Vol. 2195 (1994) pp. 584-595.
29. D. P. DeWitt, T. C. Niemoeller, C. A. Mack, and G. Yetter, “Thermal Design Methodology of Hot and Chill Plates for Photolithography,” *Integrated Circuit Metrology, Inspection, and Process Control VIII, Proc.*, SPIE Vol. 2196 (1994) pp. 432-448.
30. C. A. Mack, “Enhanced Lumped Parameter Model for Photolithography,” *Optical/Laser Microlithography VII, Proc.*, SPIE Vol. 2197 (1994) pp. 501-510.
31. C. A. Mack and E. W. Charrier, “Yield Modeling for Photolithography,” *OCG Microlithography Seminar, Interface ’94, Proc.*, (1994) pp. 171-182.
32. J. S. Petersen, C. A. Mack, J. W. Thackeray, R. Sinta, T. H. Fedynyshyn, J. M. Mori, J. D. Myers and D. A. Miller, “Characterization and Modeling of a Positive Acting Chemically Amplified Resist,” *Advances in Resist Technology and Processing XII, Proc.*, SPIE Vol. 2438 (1995) pp. 153-166.
33. J. S. Petersen, C. A. Mack, J. Sturtevant, J. D. Byers and D. A. Miller, “Non-constant Diffusion Coefficients: Short Description of Modeling and Comparison to Experimental Results,” *Advances in Resist Technology and Processing XII, Proc.*, SPIE Vol. 2438 (1995) pp. 167-180.
34. E. W. Charrier and C. A. Mack, “Yield Modeling and Enhancement for Optical Lithography,” *Optical/Laser Microlithography VIII, Proc.*, SPIE Vol. 2440 (1995) pp. 435-447.
35. C. A. Mack, “Focus Effects in Submicron Optical Lithography, Part 4: Metrics for Depth of Focus,” *Optical/Laser Microlithography VIII, Proc.*, SPIE Vol. 2440 (1995) pp. 458-471.
36. C. A. Mack and C-B. Juang, “Comparison of Scalar and Vector Modeling of Image Formation in Photoresist,” *Optical/Laser Microlithography VIII, Proc.*, SPIE Vol. 2440 (1995) pp. 381-394.

37. E. W. Charrier, C. J. Progler and C. A. Mack, "Comparison of Simulated and Experimental CD-Limited Yield for a Submicron I-Line Process," *Microelectronic Manufacturing Yield, Reliability, and Failure Analysis, Proc.*, SPIE Vol. 2635 (1995) pp. 84-94.
38. C. A. Mack, "Lithographic Effects of Acid Diffusion in Chemically Amplified Resists," *OCG Microlithography Seminar Interface '95, Proc.*, (1995) pp. 217-228.
39. C. A. Mack, T. Matsuzawa, A. Sekiguchi, Y. Minami, "Resist Metrology for Lithography Simulation, Part 1: Exposure Parameter Measurements," *Metrology, Inspection, and Process Control for Microlithography X, Proc.*, SPIE Vol. 2725 (1996) pp. 34-48.
40. A. Sekiguchi, C. A. Mack, Y. Minami, T. Matsuzawa, "Resist Metrology for Lithography Simulation, Part 2: Development Parameter Measurements," *Metrology, Inspection, and Process Control for Microlithography X, Proc.*, SPIE Vol. 2725 (1996) pp. 49-63.
41. S. H. Thornton and C. A. Mack, "Lithography Model Tuning: Matching Simulation to Experiment," *Optical Microlithography IX, Proc.*, SPIE Vol. 2726 (1996) pp. 223-235.
42. C. A. Mack, "Evaluation of Proximity Effects Using Three-Dimensional Optical Lithography Simulation," *Optical Microlithography IX, Proc.*, SPIE Vol. 2726 (1996) pp. 634-639.
43. C. A. Mack, "Reducing Proximity Effects in Optical Lithography," *Olin Microlithography Seminar Interface '96, Proc.*, (1996) pp. 325-336.
44. C. A. Mack, "Three-Dimensional Electron Beam Lithography Simulation," *Emerging Lithographic Technologies, Proc.*, SPIE Vol. 3048 (1997) pp. 76-88.
45. G. Arthur, C. A. Mack, B. Martin, "Enhancing the Development Rate Model For Optimum Simulation Capability in the Sub-Half-Micron Regime," *Advances in Resist Technology and Processing XIV, Proc.*, SPIE Vol. 3049 (1997) pp. 189-200.
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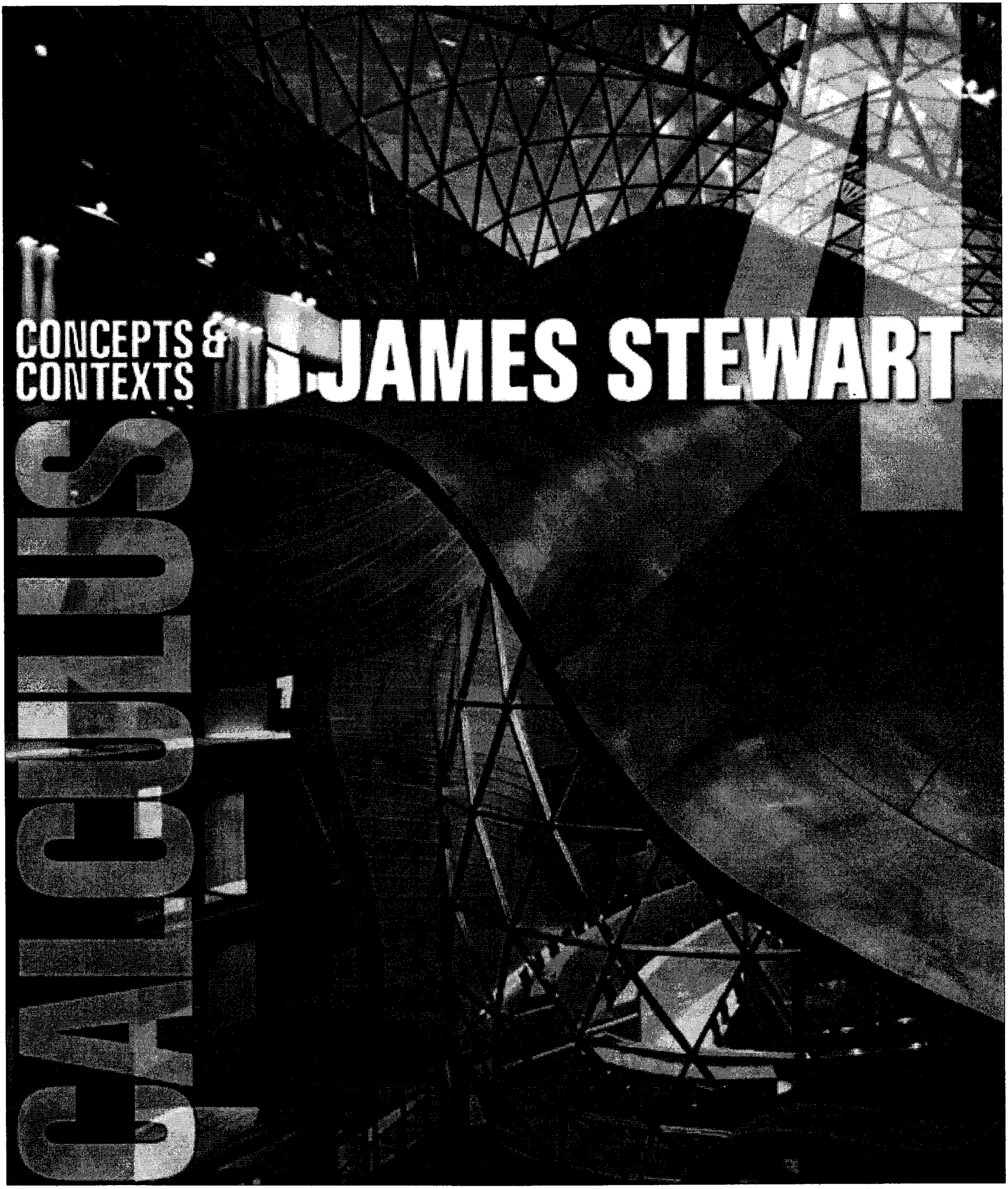
Exhibit B

Calculus (Concepts & Contexts) by James Stewart



CALCULUS

CONCEPTS & CONTEXTS

JAMES STEWART

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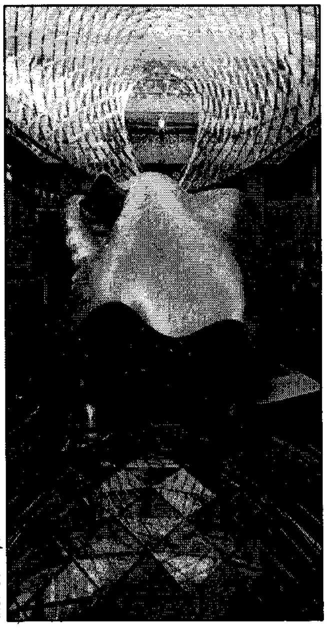

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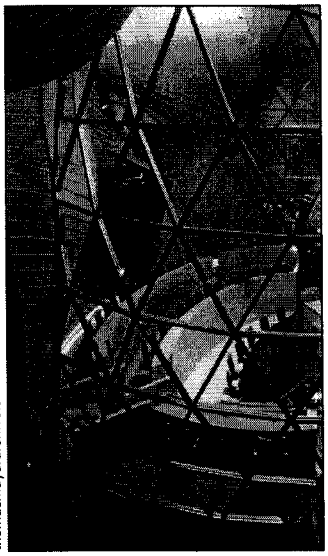

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1

Functions and Models

The fundamental objects that we deal with in calculus are functions. This chapter prepares the way for calculus by discussing the basic ideas concerning functions, their graphs, and ways of transforming and combining them. We stress that a function can be represented in different ways: by an equation, in a table, by a graph, or in words. We look at the main types of functions that occur in calculus and describe the process of using these functions as mathematical models of real-world phenomena. We also discuss the use of graphing calculators and graphing software for computers and see that parametric equations provide the best method for graphing certain types of curves.

Page(s): 11-20, Calculus (Concepts and Contexts) by James Stewart, Cengage Learning
NOOKstudy (Steve Pedersen, pedersen@stadheimgrear.com)

1.1 Four Ways to Represent a Function

Functions arise whenever one quantity depends on another. Consider the following four situations.

A. The area A of a circle depends on the radius r of the circle. The rule that connects r and A is given by the equation $A = \pi r^2$. With each positive number r there is associated one value of A, and we say that A is a *function* of r.

B. The human population of the world P depends on the time t. The table gives estimates of the world population $P(t)$ at time t, for certain years. For instance,

$$P(1950) \approx 2{,}560{,}000{,}000$$

But for each value of the time t there is a corresponding value of P, and we say that P is a function of t.

Year	Population (millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2560
1960	3040
1970	3710
1980	4450
1990	5280
2000	6080

C. The cost C of mailing a large envelope depends on the weight w of the envelope. Although there is no simple formula that connects w and C, the post office has a rule for determining C when w is known.

D. The vertical acceleration a of the ground as measured by a seismograph during an earthquake is a function of the elapsed time t. Figure 1 shows a graph generated by seismic activity during the Northridge earthquake that shook Los Angeles in 1994. For a given value of t, the graph provides a corresponding value of a.

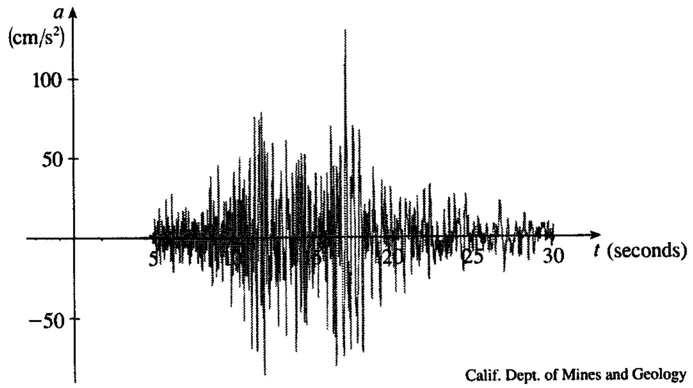

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FIGURE 1
Vertical ground acceleration during the Northridge earthquake

Each of these examples describes a rule whereby, given a number (r, t, w, or t), another number (A, P, C, or a) is assigned. In each case we say that the second number is a function of the first number.

> A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

We usually consider functions for which the sets D and E are sets of real numbers. The set D is called the **domain** of the function. The number $f(x)$ is the **value of f at x** and is read "f of x." The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain. A symbol that represents an arbitrary number in the *domain* of a function f is called an **independent variable**. A symbol that represents a number in the *range* of f is called a **dependent variable**. In Example A, for instance, r is the independent variable and A is the dependent variable.

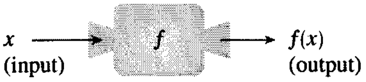

FIGURE 2
Machine diagram for a function f

It's helpful to think of a function as a **machine** (see Figure 2). If x is in the domain of the function f, then when x enters the machine, it's accepted as an input and the machine produces an output $f(x)$ according to the rule of the function. Thus we can think of the domain as the set of all possible inputs and the range as the set of all possible outputs.

The preprogrammed functions in a calculator are good examples of a function as a machine. For example, the square root key on your calculator computes such a function. You press the key labeled $\sqrt{\ }$ (or $\sqrt{x}$) and enter the input x. If $x < 0$, then x is not in the domain of this function; that is, x is not an acceptable input, and the calculator will indicate an error. If $x \geqslant 0$, then an *approximation* to $\sqrt{x}$ will appear in the display. Thus the $\sqrt{x}$ key on your calculator is not quite the same as the exact mathematical function f defined by $f(x) = \sqrt{x}$.

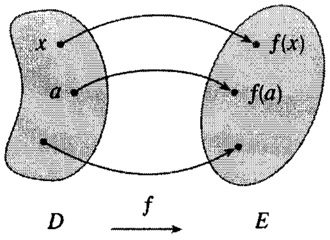

FIGURE 3
Arrow diagram for f

Another way to picture a function is by an **arrow diagram** as in Figure 3. Each arrow connects an element of D to an element of E. The arrow indicates that $f(x)$ is associated with x, $f(a)$ is associated with a, and so on.

The most common method for visualizing a function is its graph. If f is a function with domain D, then its **graph** is the set of ordered pairs

$$\{(x, f(x)) \mid x \in D\}$$

(Notice that these are input-output pairs.) In other words, the graph of f consists of all points (x, y) in the coordinate plane such that $y = f(x)$ and x is in the domain of f.

The graph of a function f gives us a useful picture of the behavior or "life history" of a function. Since the y-coordinate of any point (x, y) on the graph is $y = f(x)$, we can read the value of $f(x)$ from the graph as being the height of the graph above the point x (see Figure 4). The graph of f also allows us to picture the domain of f on the x-axis and its range on the y-axis as in Figure 5.

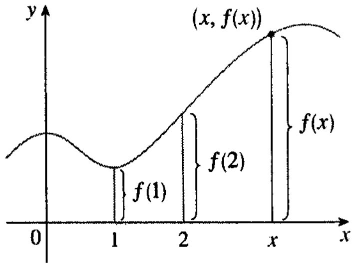

FIGURE 4

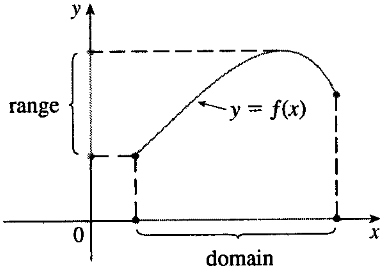

FIGURE 5

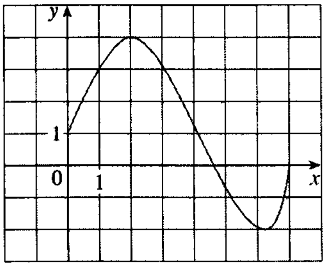

FIGURE 6

The notation for intervals is given in Appendix A.

EXAMPLE 1 **Reading information from a graph** The graph of a function f is shown in Figure 6.

(a) Find the values of $f(1)$ and $f(5)$.
(b) What are the domain and range of f?

SOLUTION

(a) We see from Figure 6 that the point $(1, 3)$ lies on the graph of f, so the value of f at 1 is $f(1) = 3$. (In other words, the point on the graph that lies above $x = 1$ is 3 units above the x-axis.)

When $x = 5$, the graph lies about 0.7 unit below the x-axis, so we estimate that $f(5) \approx -0.7$.

(b) We see that $f(x)$ is defined when $0 \leqslant x \leqslant 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4, so the range of f is

$$\{y \mid -2 \leqslant y \leqslant 4\} = [-2, 4]$$

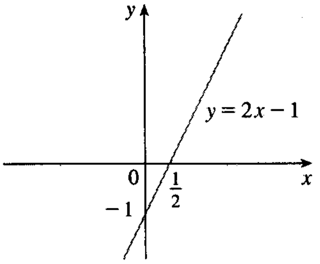

FIGURE 7

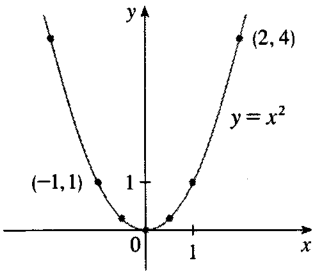

FIGURE 8

EXAMPLE 2 Sketch the graph and find the domain and range of each function.

(a) $f(x) = 2x - 1$ (b) $g(x) = x^2$

SOLUTION

(a) The equation of the graph is $y = 2x - 1$, and we recognize this as being the equation of a line with slope 2 and y-intercept -1. (Recall the slope-intercept form of the equation of a line: $y = mx + b$. See Appendix B.) This enables us to sketch a portion of the graph of f in Figure 7. The expression $2x - 1$ is defined for all real numbers, so the domain of f is the set of all real numbers, which we denote by $\mathbb{R}$. The graph shows that the range is also $\mathbb{R}$.

(b) Since $g(2) = 2^2 = 4$ and $g(-1) = (-1)^2 = 1$, we could plot the points $(2, 4)$ and $(-1, 1)$, together with a few other points on the graph, and join them to produce the graph (Figure 8). The equation of the graph is $y = x^2$, which represents a parabola (see Appendix B). The domain of g is $\mathbb{R}$. The range of g consists of all values of $g(x)$, that is, all numbers of the form x^2. But $x^2 \geqslant 0$ for all numbers x and any positive number y is a square. So the range of g is $\{y \mid y \geqslant 0\} = [0, \infty)$. This can also be seen from Figure 8.

EXAMPLE 3 Evaluating a difference quotient

If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a + h) - f(a)}{h}$.

SOLUTION We first evaluate $f(a + h)$ by replacing x by $a + h$ in the expression for $f(x)$:

$$\begin{aligned} f(a + h) &= 2(a + h)^2 - 5(a + h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 5(a + h) + 1 \\ &= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1 \end{aligned}$$

Then we substitute into the given expression and simplify:

$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5 \end{aligned}$$

The expression

$$\frac{f(a + h) - f(a)}{h}$$

in Example 3 is called a **difference quotient** and occurs frequently in calculus. As we will see in Chapter 2, it represents the average rate of change of $f(x)$ between $x = a$ and $x = a + h$.

Representations of Functions

There are four possible ways to represent a function:

▪ verbally	(by a description in words)
▪ numerically	(by a table of values)
▪ visually	(by a graph)
▪ algebraically	(by an explicit formula)

If a single function can be represented in all four ways, it's often useful to go from one representation to another to gain additional insight into the function. (In Example 2, for instance, we started with algebraic formulas and then obtained the graphs.) But certain functions are described more naturally by one method than by another. With this in mind, let's reexamine the four situations that we considered at the beginning of this section.

A. The most useful representation of the area of a circle as a function of its radius is probably the algebraic formula $A(r) = \pi r^2$, though it is possible to compile a table of values or to sketch a graph (half a parabola). Because a circle has to have a positive radius, the domain is $\{r \mid r > 0\} = (0, \infty)$, and the range is also $(0, \infty)$.

Year	Population (millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2560
1960	3040
1970	3710
1980	4450
1990	5280
2000	6080

B. We are given a description of the function in words: $P(t)$ is the human population of the world at time t. The table of values of world population provides a convenient representation of this function. If we plot these values, we get the graph (called a *scatter plot*) in Figure 9. It too is a useful representation; the graph allows us to absorb all the data at once. What about a formula? Of course, it's impossible to devise an explicit formula that gives the exact human population $P(t)$ at any time t. But it is possible to find an expression for a function that *approximates* $P(t)$. In fact, using methods explained in Section 1.5, we obtain the approximation

$$P(t) \approx f(t) = (0.008079266) \cdot (1.013731)^t$$

and Figure 10 shows that it is a reasonably good "fit." The function f is called a *mathematical model* for population growth. In other words, it is a function with an explicit formula that approximates the behavior of our given function. We will see, however, that the ideas of calculus can be applied to a table of values; an explicit formula is not necessary.

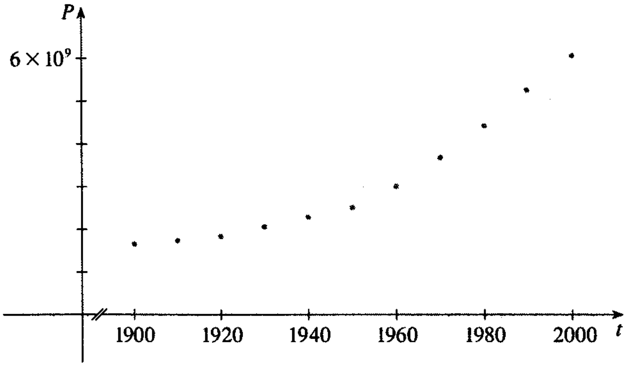

FIGURE 9

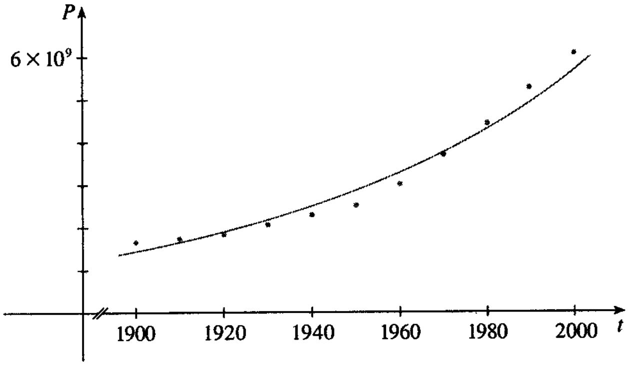

FIGURE 10

The function P is typical of the functions that arise whenever we attempt to apply calculus to the real world. We start with a verbal description of a function. Then we may be able to construct a table of values of the function, perhaps from instrument readings in a scientific experiment. Even though we don't have complete knowledge of the values of the function, we will see throughout the book that it is still possible to perform the operations of calculus on such a function.

A function defined by a table of values is called a *tabular* function.

w (ounces)	$C(w)$ (dollars)
$0 < w \leqslant 1$	0.83
$1 < w \leqslant 2$	1.00
$2 < w \leqslant 3$	1.17
$3 < w \leqslant 4$	1.34
$4 < w \leqslant 5$	1.51
⋮	⋮
$12 < w \leqslant 13$	2.87

C. Again the function is described in words: $C(w)$ is the cost of mailing a large envelope with weight w. The rule that the US Postal Service used as of 2008 is as follows: The cost is 83 cents for up to 1 oz, plus 17 cents for each additional ounce (or less) up to 13 oz. The table of values shown in the margin is the most convenient representation for this function, though it is possible to sketch a graph (see Example 10).

D. The graph shown in Figure 1 is the most natural representation of the vertical acceleration function $a(t)$. It's true that a table of values could be compiled, and it is even possible to devise an approximate formula. But everything a geologist needs to know—amplitudes and patterns—can be seen easily from the graph. (The same is true for the patterns seen in electrocardiograms of heart patients and polygraphs for lie-detection.)

Page(s): 11-20, Calculus (Concepts and Contexts) by James Stewart, Cengage Learning
NOOKstudy (Steve Pedersen, pedersen@stadheimgrear.com)

EXAMPLE 4 Drawing a graph from a verbal description When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running. Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.

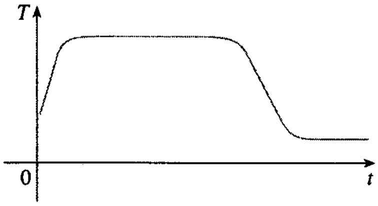

FIGURE 11

SOLUTION The initial temperature of the running water is close to room temperature because the water has been sitting in the pipes. When the water from the hot-water tank starts flowing from the faucet, T increases quickly. In the next phase, T is constant at the temperature of the heated water in the tank. When the tank is drained, T decreases to the temperature of the water supply. This enables us to make the rough sketch of T as a function of t in Figure 11. ■

In the following example we start with a verbal description of a function in a physical situation and obtain an explicit algebraic formula. The ability to do this is a useful skill in solving calculus problems that ask for the maximum or minimum values of quantities.

V EXAMPLE 5 Expressing a cost as a function A rectangular storage container with an open top has a volume of 10 m^3. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

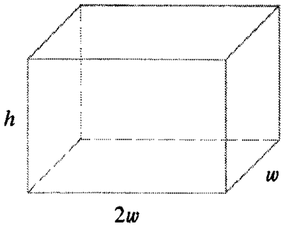

FIGURE 12

SOLUTION We draw a diagram as in Figure 12 and introduce notation by letting w and $2w$ be the width and length of the base, respectively, and h be the height.

The area of the base is $(2w)w = 2w^2$, so the cost, in dollars, of the material for the base is $10(2w^2)$. Two of the sides have area wh and the other two have area $2wh$, so the cost of the material for the sides is $6[2(wh) + 2(2wh)]$. The total cost is therefore

$$C = 10(2w^2) + 6[2(wh) + 2(2wh)] = 20w^2 + 36wh$$

To express C as a function of w alone, we need to eliminate h and we do so by using the fact that the volume is 10 m^3. Thus

$$w(2w)h = 10$$

which gives

$$h = \frac{10}{2w^2} = \frac{5}{w^2}$$

Substituting this into the expression for C, we have

$$C = 20w^2 + 36w\left(\frac{5}{w^2}\right) = 20w^2 + \frac{180}{w}$$

Therefore the equation

$$C(w) = 20w^2 + \frac{180}{w} \qquad w > 0$$

expresses C as a function of w. ■

PS In setting up applied functions as in Example 5, it may be useful to review the principles of problem solving as discussed on page 83, particularly *Step 1: Understand the Problem.*

EXAMPLE 6 Find the domain of each function.

(a) $f(x) = \sqrt{x + 2}$ (b) $g(x) = \dfrac{1}{x^2 - x}$

SOLUTION

(a) Because the square root of a negative number is not defined (as a real number), the domain of f consists of all values of x such that $x + 2 \geqslant 0$. This is equivalent to $x \geqslant -2$, so the domain is the interval $[-2, \infty)$.

Domain Convention

If a function is given by a formula and the domain is not stated explicitly, the convention is that the domain is the set of all numbers for which the formula makes sense and defines a real number.

(b) Since

$$g(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

and division by 0 is not allowed, we see that $g(x)$ is not defined when $x = 0$ or $x = 1$. Thus the domain of g is

$$\{x \mid x \neq 0, x \neq 1\}$$

which could also be written in interval notation as

$$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$$

The graph of a function is a curve in the xy-plane. But the question arises: Which curves in the xy-plane are graphs of functions? This is answered by the following test.

The Vertical Line Test A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

The reason for the truth of the Vertical Line Test can be seen in Figure 13. If each vertical line $x = a$ intersects a curve only once, at (a, b), then exactly one functional value is defined by $f(a) = b$. But if a line $x = a$ intersects the curve twice, at (a, b) and (a, c), then the curve can't represent a function because a function can't assign two different values to a.

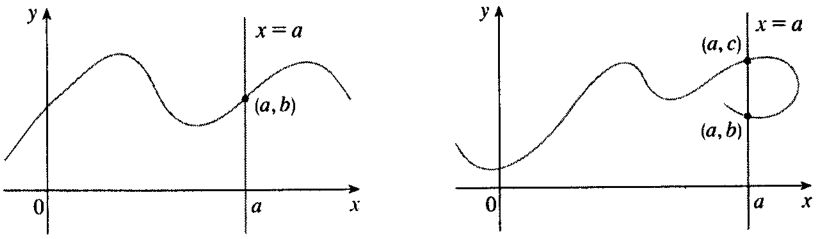

FIGURE 13

For example, the parabola $x = y^2 - 2$ shown in Figure 14(a) is not the graph of a function of x because, as you can see, there are vertical lines that intersect the parabola twice. The parabola, however, does contain the graphs of *two* functions of x. Notice that the equation $x = y^2 - 2$ implies $y^2 = x + 2$, so $y = \pm\sqrt{x+2}$. Thus the upper and lower halves of the parabola are the graphs of the functions $f(x) = \sqrt{x+2}$ [from Example 6(a)] and $g(x) = -\sqrt{x+2}$. [See Figures 14(b) and (c).] We observe that if we reverse the roles of x and y, then the equation $x = h(y) = y^2 - 2$ *does* define x as a function of y (with y as the independent variable and x as the dependent variable) and the parabola now appears as the graph of the function h.

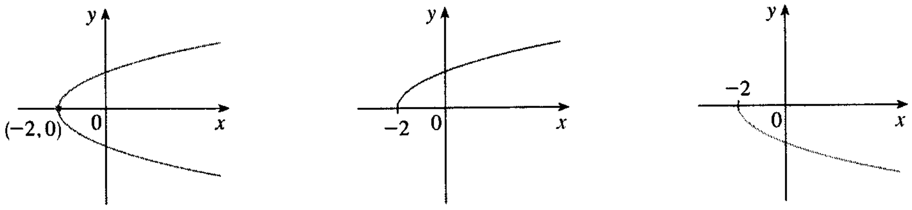

FIGURE 14 (a) $x = y^2 - 2$ (b) $y = \sqrt{x+2}$ (c) $y = -\sqrt{x+2}$

Piecewise Defined Functions

The functions in the following four examples are defined by different formulas in different parts of their domains.

V **EXAMPLE 7** **Graphing a piecewise defined function** A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leqslant 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

Evaluate $f(0)$, $f(1)$, and $f(2)$ and sketch the graph.

SOLUTION Remember that a function is a rule. For this particular function the rule is the following: First look at the value of the input x. If it happens that $x \leqslant 1$, then the value of $f(x)$ is $1 - x$. On the other hand, if $x > 1$, then the value of $f(x)$ is x^2.

$$\text{Since } 0 \leqslant 1, \text{ we have } f(0) = 1 - 0 = 1.$$

$$\text{Since } 1 \leqslant 1, \text{ we have } f(1) = 1 - 1 = 0.$$

$$\text{Since } 2 > 1, \text{ we have } f(2) = 2^2 = 4.$$

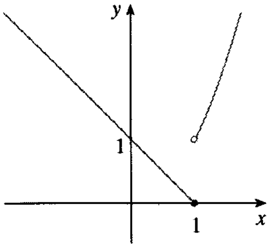

FIGURE 15

How do we draw the graph of f? We observe that if $x \leqslant 1$, then $f(x) = 1 - x$, so the part of the graph of f that lies to the left of the vertical line $x = 1$ must coincide with the line $y = 1 - x$, which has slope -1 and y-intercept 1. If $x > 1$, then $f(x) = x^2$, so the part of the graph of f that lies to the right of the line $x = 1$ must coincide with the graph of $y = x^2$, which is a parabola. This enables us to sketch the graph in Figure 15. The solid dot indicates that the point $(1, 0)$ is included on the graph; the open dot indicates that the point $(1, 1)$ is excluded from the graph.

The next example of a piecewise defined function is the absolute value function. Recall that the **absolute value** of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line. Distances are always positive or 0, so we have

For a more extensive review of absolute values, see Appendix A.

$$|a| \geqslant 0 \qquad \text{for every number } a$$

For example,

$$|3| = 3 \qquad |-3| = 3 \qquad |0| = 0 \qquad |\sqrt{2} - 1| = \sqrt{2} - 1 \qquad |3 - \pi| = \pi - 3$$

In general, we have

$$\boxed{\begin{aligned} |a| &= a && \text{if } a \geqslant 0 \\ |a| &= -a && \text{if } a < 0 \end{aligned}}$$

(Remember that if a is negative, then $-a$ is positive.)

EXAMPLE 8 Sketch the graph of the absolute value function $f(x) = |x|$.

SOLUTION From the preceding discussion we know that

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases}$$

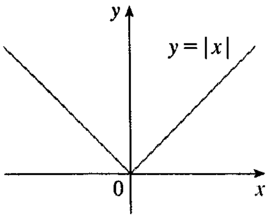

FIGURE 16

Using the same method as in Example 7, we see that the graph of f coincides with the line $y = x$ to the right of the y-axis and coincides with the line $y = -x$ to the left of the y-axis (see Figure 16).

EXAMPLE 9 Find a formula for the function f graphed in Figure 17.

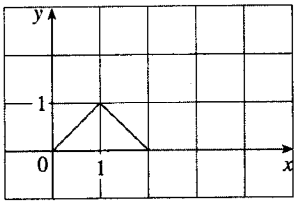

FIGURE 17

SOLUTION The line through $(0, 0)$ and $(1, 1)$ has slope $m = 1$ and y-intercept $b = 0$, so its equation is $y = x$. Thus, for the part of the graph of f that joins $(0, 0)$ to $(1, 1)$, we have

$$f(x) = x \qquad \text{if } 0 \leqslant x \leqslant 1$$

Point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

See Appendix B.

The line through $(1, 1)$ and $(2, 0)$ has slope $m = -1$, so its point-slope form is

$$y - 0 = (-1)(x - 2) \qquad \text{or} \qquad y = 2 - x$$

So we have

$$f(x) = 2 - x \qquad \text{if } 1 < x \leqslant 2$$

We also see that the graph of f coincides with the x-axis for $x > 2$. Putting this information together, we have the following three-piece formula for f:

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } 1 < x \leqslant 2 \\ 0 & \text{if } x > 2 \end{cases}$$

EXAMPLE 10 **Graph of a postage function** In Example C at the beginning of this section we considered the cost $C(w)$ of mailing a large envelope with weight w. In effect, this is a piecewise defined function because, from the table of values, we have

$$C(w) = \begin{cases} 0.83 & \text{if } 0 < w \leqslant 1 \\ 1.00 & \text{if } 1 < w \leqslant 2 \\ 1.17 & \text{if } 2 < w \leqslant 3 \\ 1.34 & \text{if } 3 < w \leqslant 4 \\ \vdots & \end{cases}$$

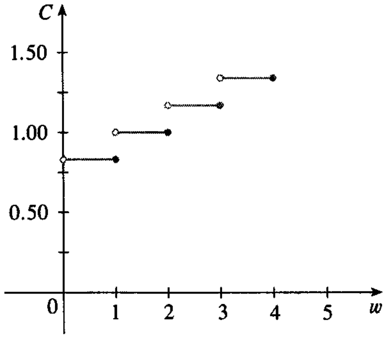

FIGURE 18

The graph is shown in Figure 18. You can see why functions similar to this one are called **step functions**—they jump from one value to the next. Such functions will be studied in Chapter 2.

Symmetry

If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an **even function**. For instance, the function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

The geometric significance of an even function is that its graph is symmetric with respect to the y-axis (see Figure 19). This means that if we have plotted the graph of f for $x \geqslant 0$, we obtain the entire graph simply by reflecting this portion about the y-axis.

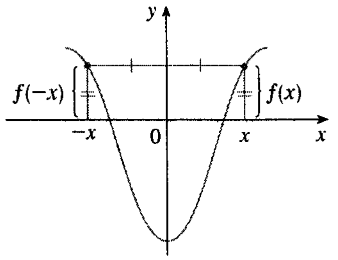

FIGURE 19 An even function

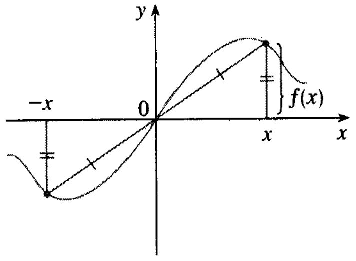

FIGURE 20 An odd function

If f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd function**. For example, the function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

The graph of an odd function is symmetric about the origin (see Figure 20). If we already have the graph of f for $x \geqslant 0$, we can obtain the entire graph by rotating this portion through 180° about the origin.

V **EXAMPLE 11** Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$ (b) $g(x) = 1 - x^4$ (c) $h(x) = 2x - x^2$

SOLUTION

(a)
$$\begin{aligned} f(-x) &= (-x)^5 + (-x) = (-1)^5x^5 + (-x) \\ &= -x^5 - x = -(x^5 + x) \\ &= -f(x) \end{aligned}$$

Therefore f is an odd function.

(b)
$$g(-x) = 1 - (-x)^4 = 1 - x^4 = g(x)$$

So g is even.

(c)
$$h(-x) = 2(-x) - (-x)^2 = -2x - x^2$$

Since $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$, we conclude that h is neither even nor odd.

The graphs of the functions in Example 11 are shown in Figure 21. Notice that the graph of h is symmetric neither about the y-axis nor about the origin.

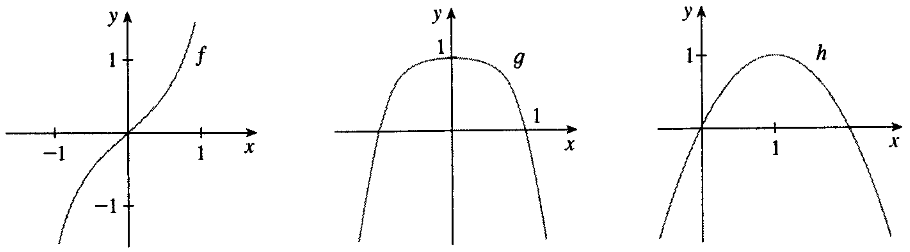

FIGURE 21 (a) (b) (c)

Calculus

Concepts and Contexts | 4e

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Increasing and Decreasing Functions

The graph shown in Figure 22 rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be increasing on the interval $[a, b]$, decreasing on $[b, c]$, and increasing again on $[c, d]$. Notice that if x_1 and x_2 are any two numbers between a and b with $x_1 < x_2$, then $f(x_1) < f(x_2)$. We use this as the defining property of an increasing function.

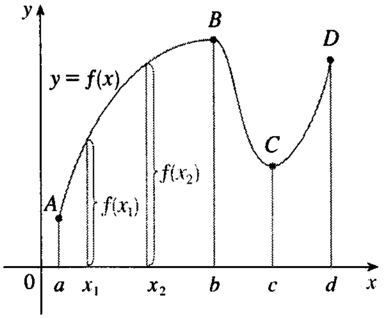


FIGURE 22

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

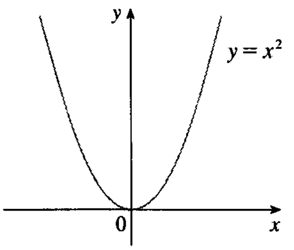


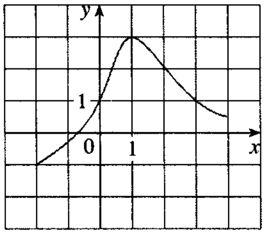
FIGURE 23

In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for *every* pair of numbers x_1 and x_2 in I with $x_1 < x_2$.

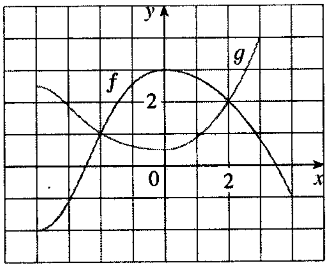
You can see from Figure 23 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

1.1 Exercises

1. The graph of a function f is given.
   (a) State the value of $f(1)$.
   (b) Estimate the value of $f(-1)$.
   (c) For what values of x is $f(x) = 1$?
   (d) Estimate the value of x such that $f(x) = 0$.
   (e) State the domain and range of f.
   (f) On what interval is f increasing?



2. The graphs of f and g are given.
   (a) State the values of $f(-4)$ and $g(3)$.
   (b) For what values of x is $f(x) = g(x)$?
   (c) Estimate the solution of the equation $f(x) = -1$.
   (d) On what interval is f decreasing?
   (e) State the domain and range of f.
   (f) State the domain and range of g.



3. Figure 1 was recorded by an instrument operated by the California Department of Mines and Geology at the University Hospital of the University of Southern California in Los Angeles. Use it to estimate the range of the vertical ground acceleration function at USC during the Northridge earthquake.

4. In this section we discussed examples of ordinary, everyday functions: Population is a function of time, postage cost is a function of weight, water temperature is a function of time. Give three other examples of functions from everyday life that are described verbally. What can you say about the domain and range of each of your functions? If possible, sketch a rough graph of each function.

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1. Homework Hints available in TEC